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EFFECT OF LOAD ON MACHINE GEOMETRY WITH RESPECT TO THE  
WEIGHT AND TORQUE

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Ce mémoire intitulé:

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TORQUE

présenté par : ASADISHAD Behkam

en vue de l'obtention du diplôme de : Maîtrise ès sciences appliquées

a été dûment accepté par le jury d'examen constitué de :

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## **DEDICATION**

I would like to dedicate this thesis to my family. I am very thankful for all of your support and encouragement during my studies.



## **ACKNOWLEDGEMENT**

I would like to acknowledge my advisor, René Mayer, for his invaluable guidance, supports, and encouragement. It has been an honor to work with him. I thank Mechanical Engineering faculty and technicians in Manufacturing Laboratory, Guy Gironne and Vincent Mayer for their companionship. I also thank my family, for without their support this degree would never have been possible.

I would also like to acknowledge the funding research association, Recherche et Innovation Synergétiques en Aérospatiale (CRIAQ), for supporting my research.

## RÉSUMÉ

Dans cette étude, une méthode est développée pour estimer et prédire les coefficients d'erreur de la déformation d'une machine CNC à cinq axes sous différentes conditions de chargement. Pour étudier l'effet de différents poids et les couples sur les erreurs de la machine, les blocs lourds identiques (10 kg chacun) ainsi que des entretoises lumière ont été conçus et fabriqués. Après installer les blocs et les boules sur la palette, la machine sonde les boules pour différents angles indexés par rapport à ses deux axes de rotation (B et C axes) en utilisant les codes G générés par programme RUMBA (the 3D reconfigurable uncalibrated master balls artifact). Au total, il y a 26 types d'erreurs et chaque erreur a maximum de cinq coefficients qui sont appelés coefficients d'erreur estimés. Les données obtenues à partir de la machine sont traitées afin d'estimer les valeurs des coefficients d'erreur pour chaque erreur. Ces derniers permettent d'obtenir les graphiques polynômes de chaque erreur en utilisant leurs coefficients d'erreur. En outre, la matrice de corrélation de Pearson est obtenue pour tous les coefficients d'erreur et des poids et des couples. Les éléments de la matrice de corrélation (entre -1 et 1) indiquent le degré de dépendance de chaque coefficient d'erreur pour les autres coefficients, ainsi que le poids et le couple. Enfin, le procédé d'ajustement de la courbe est utilisé pour modéliser chaque coefficient d'erreur en fonction du poids et du couple. Ceci permet de prédire la valeur du coefficient d'erreur pour toutes les valeurs de poids et de couple supprimant la nécessité l'essai expérimental. L'analyse résiduelle (la différence entre les valeurs expérimentales et les valeurs prédites) est ensuite utilisée pour vérifier l'exactitude du modèle. Les résultats de cette étude montrent que cette méthode est suffisamment précise pour estimer les erreurs de la machines qui est testés (HU40T machine CNC à cinq axes). Le modèle nous permet d'estimer les valeurs des coefficients d'erreur en fonction du poids uniquement, en fonction du couple uniquement et en fonction du poids et du couple simultanément. Cette connaissance est importante pour la compensation d'erreur ou la suppression d'erreur dans la machine pour augmenter la précision de la fabrication.

## ABSTRACT

In this study, a method is developed to estimate and predict the error coefficients of a five-axis CNC machine tool deformation under different loading conditions. To study the effect of different weights and torques on the machine errors, identical heavy blocks (10 Kilograms each) as well as light spacers (less than 1 Kilogram) are designed and fabricated. After assembling the blocks and master balls on the pallet, the machine probes the balls for different indexations from the rotary axes (B and C axes) of the machine using G-codes generated from the 3D reconfigurable uncalibrated master balls artefact (RUMBA) program. In total, there are 26 types of errors and each error has maximum five coefficients which are called estimated error coefficients. The raw data obtained from machine is processed to estimate the values of the error coefficients for each error. Then, the polynomial function of each error can be achieved by using their error coefficients. Furthermore, Pearson's correlation matrix is obtained for all error coefficients and weights and torques. The elements of the correlation matrix (between -1 and 1) show the extent of dependency of each error coefficient to other coefficients as well as to the weight and torque. Finally, the curve fitting method is used to model each error coefficient as a function of weight and torque. The modeling allows predicting the value of the error coefficient for any weights and torques without doing an experimental test. Residual analysis (difference between the experimental and the predicted values) is then used to verify the accuracy of the modeling. The results of this study show that this method is accurate enough to estimate the errors of the tested machine tool (HU40T 5-axis CNC machine tools). The model enables us to estimate the error coefficient values as a function of weight independently, torque independently and weight and torque simultaneously. This knowledge is important for error compensation or even error removal in machine to increase the precision of the manufacturing.

## TABLE OF CONTENTS

DEDICATION.....	iii
ACKNOWLEDGEMENT.....	iv
RÉSUMÉ.....	v
ABSTRACT.....	vi
TABLE OF CONTENTS.....	vii
LIST OF TABLES.....	x
LIST OF FIGURES.....	xi
LIST OF APPENDICES.....	xv
CHAPTER 1. INTRODUCTION.....	1
1.1 Design and build means to apply controlled loading conditions.....	3
1.2 Determine the ball location for the RUMBA method and write appropriate G-code.....	4
1.3 Conduct the test.....	4
1.4 Analyze the error to identify possible patterns.....	5
1.5 Represent each error as a function of loading.....	5
CHAPTER 2. LOADING FIXTURES.....	6
2.1 Design stack.....	6
2.2 Assemble the blocks.....	8
2.3 Stack deflection.....	11
CHAPTER 3. RUMBA BALL LOCATION AND G-CODE.....	13
3.1 The mathematical model.....	14
3.1.1 Mathematical modeling of a joint motion.....	15
3.1.2 Mathematical modeling of a link.....	17
3.1.3 Nominal modeling.....	19
3.1.4 Exact modeling.....	20

3.2	Generating G-code.....	23
CHAPTER 4. CONDUCT THE TEST.....		26
4.1	Machine tool.....	26
4.2	Master ball.....	28
4.3	Probe.....	29
4.4	Block.....	31
4.5	Spacer.....	31
4.6	Set of axes indexations.....	32
4.7	Validation test.....	36
CHAPTER 5. ANALYZE THE ERROR DATA.....		39
5.1	Polynomial graphs.....	39
5.2	Discrepancy of the errors.....	68
5.3	Volumetric Errors.....	74
CHAPTER 6. ERROR MODELING.....		76
6.1	Correlation.....	77
6.1.1	Correlation between the error coefficients.....	79
6.1.2	Correlation between the error coefficients and weight and torque.....	80
6.2	Curve fitting.....	81
6.2.1	Model the error coefficients as only a function of weight only.....	82
6.2.2	Model the error coefficients as only a function of torque only.....	84
6.2.3	Model the error coefficients as a function of weight and torque.....	86
6.3	Goodness-of-Fit Statistics (residual analysis).....	88
CONCLUSION.....		94
	Polynomial graphs.....	94
	Correlation.....	95
	Curve fitting.....	95
	Residual analysis.....	96

REFERENCES.....	98
APPENDICES.....	101

## LIST OF TABLES

Table 2-1: The tests specifications.....	10
Table 2-2: The values of maximum deflection and the angle of the deflection for all tests.....	11
Table 4-1: Standard Features of the machine tool.....	26
Table 4-2: The probe components.....	29
Table 4-3: The nominal master ball coordinates for different tests.....	34
Table 6-1: All the possible error coefficients.....	76
Table 6-2: The 83 estimated error coefficients.....	77
Table 6-3: Correlation matrix for weight and torque.....	80
Table B-1: The estimated value of the error coefficients.....	122
Table D-1: The elements of the Jacobian matrix.....	132
Table E-1: Correlation matrix.....	134
Table F-1_a: The elements of the matrix PA from EXX2 to ECZ1.....	152
Table F-1_b: The elements of the matrix PA from ECZ2 to ECC4.....	153
Table F-2_a: The elements of the matrix PB from EXX2 to ECZ1.....	154
Table F-2_b: The elements of the matrix PB from ECZ2 to ECC4.....	155
Table F-3_a: The elements of the matrix PB from EXX2 to EAB1.....	156
Table F-3_b: The elements of the matrix PB from EAB2 to ECC4.....	157

## LIST OF FIGURES

Figure 2-1: Block.....	6
Figure 2-2: Spacer.....	6
Figure 2-3: The fully dimensioned block.....	7
Figure 2-4: The fully dimensioned spacer.....	7
Figure 2-5: Different assembly of tests.....	8
Figure 2-6: The maximum deflection of the blocks for each test.....	12
Figure 2-7: The angle of the deflection for each test.....	12
Figure 3-1: Master ball artefac.....	13
Figure 3-2: Probe.....	14
Figure 3-3: Mathematical modeling of a joint motion.....	16
Figure 3-4_a: Modeling of the link and its parameters.....	17
Figure 3-4_b: Modeling of the link and its axis location errors.....	18
Figure 3-5: The nominal kinematic chain of the 5-axis CNC machine tool.....	20
Figure 3-6: The exact kinematic chain of the 5-axis CNC machine tool.....	21
Figure3-7: Machine pallet position in $B=0^\circ, C=0^\circ$ .....	24
Figure3-8: Machine pallet position in $B=-90^\circ, C=0^\circ$ .....	24
Figure 3-9: The flowchart for generating G-code in RUMBA.....	25
Figure4-1: MITSUI SEIKI 5-Axis High Production Machining Center Model HU40T.....	28
Figure 4-2: The artefact master ball.....	28
Figure 4-3: The MP 700 Renishaw probe.....	29



Figure 4-4: Gildemeister-devlieg system-werkzeuge gmbh microset.....	30
Figure 4-5: The fabricated block.....	31
Figure 4-6: The fabricated spacer.....	31
Figure 4-7: The position of B and C rotary axes of the machine pallet .....	32
Figure 4-8: 15 Of 19 axes indexations.....	33
Figure 4-9_a: Diagonal pattern of ball locations.....	34
Figure 4-9_b: 4 corners pattern of ball location.....	34
Figure 4-10: The validation test set up with only blocks.....	36
Figure 4-11: The validation test set up with blocks and spacers.....	37
Figure 4-12: The length variation of the magnetic ball bar Renishaw.....	37
Figure 4-13: The 2D schematic of the validation test.....	38
Figure 5-1: Different polynomial graphs.....	39
Figure 5-2: The rainbow colors of the polynomial graphs.....	41
Figure 5-3: The polynomial graphs of the error EXX.....	42
Figure 5-4: The polynomial graphs of the error EYX.....	43
Figure 5-5: The polynomial graphs of the error EZX.....	44
Figure 5-6: the polynomial graphs of the error EAX.....	45
Figure 5-7: The polynomial graphs of the error EBX.....	46
Figure 5-8: The polynomial graphs of the error ECX.....	47
Figure 5-9: The polynomial graphs of the error EXY.....	48
Figure 5-10: The polynomial graphs of the error EYY.....	49

Figure 5-11: the polynomial graphs of the error EZY.....	50
Figure 5-12: The polynomial graphs of the error EXZ.....	51
Figure 5-13: The polynomial graphs of the error EYZ.....	52
Figure 5-14: The polynomial graphs of the error EZZ.....	53
Figure 5-15: The polynomial graphs of the error EAZ.....	54
Figure 5-16: The polynomial graphs of the error ECZ.....	55
Figure 5-17: The polynomial graphs of the error EXB.....	56
Figure 5-18: The polynomial graphs of the error EYB.....	57
Figure 5-19: The polynomial graphs of the error EZB.....	58
Figure 5-20: The polynomial graphs of the error EAB.....	59
Figure 5-21: The polynomial graphs of the error EBB.....	60
Figure 5-22: The polynomial graphs of the error ECB.....	61
Figure 5-23: The polynomial graphs of the error EXC.....	62
Figure 5-24: The polynomial graphs of the error EYC.....	63
Figure 5-25: The polynomial graphs of the error EZC.....	64
Figure 5-26: The polynomial graphs of the error EAC.....	65
Figure 5-27: The polynomial graphs of the error EBC.....	66
Figure 5-28: The polynomial graphs of the error ECC.....	67
Figure 5-29: Discrepancy of the errors in different tests.....	69
Figure 5-30: The maximum volumetric error.....	75
Figure 5-31: The mean volumetric error.....	75

Figure 6-1: The error coefficients modeling as a function of weight.....	82
Figure 6-2: The error coefficients modeling as a function of torque.....	84
Figure 6-3: The error coefficients modeling as a function of weight and torque.....	87
Figure 6-4: The data fit and the residual graph for Fit 1.....	88
Figure 6-5: The data fit and the residual graph for Fit 2.....	89
Figure 6-6: The data fit and the residual graph for Fit 3.....	90
Figure 6-7-a: The residual analysis for error EYX2.....	92
Figure 6-7-b: The residual analysis for error EXB1.....	92
Figure 6-7-c: The residual analysis for error ECC3.....	93
Figure G-1: The error modeling as a function of weight and torque.....	158

## LIST OF APPENDICES

Appendix I: Matlab programs.....	101
Appendix II: The estimated value of the error coefficients.....	121
Appendix III: The equations of nominal modeling, exact modeling and volumetric errors.....	123
Appendix IV: The elements of the Jacobian matrix.....	131
Appendix V: Correlation matrix.....	133
Appendix VI: The coefficients of the fitted curves.....	151
Appendix VII: The error modeling as a function of weight and torque.....	157

## Chapter 1 Introduction

Five axes CNC machining is very popular in industry for fabricating complex parts in aerospace and other precision fields. In these machine tools, there are three linear axes (X, Y and Z axis) and two angular axes (the rotation of the pallet and the rotation of the cutting tool). The finished parts which have been machined by five-axis CNC machine tools, have smooth surface with high quality and high accuracy [1,2]. However, the presence of two rotary axes can increase the number of error parameters and decrease the machine calibration, especially when the direction of cutting tool is not parallel to the gravity. These kinds of machine tools are called Horizontal. Their cutting tool is horizontal and thus, the workpiece, the pallet, and the cutting tool are effected by their weights [3].

There are numerous studies regarding machine tools errors. In some studies, the circular test method has been used to measure the errors and diagnose the source of errors. For instance, Zargarbashi and Mayer assessed axis motion error using magnetic double ball bar[4]. In another study, Knapp used a new device called R-test to measure X-Y-Z deviations of circular and spherical movements to calibrate parameter errors such as backlash, positioning error, squareness and parallelism [5]. Hong et al presented a method to identify the motion error sources (non-directional error pattern and directional error pattern) by using circular test [6]. Some other studies investigated the accuracy of the machine tools. For instance, Lei and Hsu presented an error model in the template of homogenous transformation matrix (HTM) and did accuracy tests on a 5-axes CNC machine tool with 3D probe-ball [7,8]. Lu et al presented a new two-dimensional method to investigate the existence of radial error motion in the spindle [9,10]. Iwai and Mitsui developed a method to determine 11 error parameters created by the link length and the inclination of the link axes using rotary encoders and link mechanisms [11]. Yang developed a multi-probe measurement system to investigate the motion accuracy of high –precision micro-coordinate measuring machine [12]. In some studies, the laser interferometer has been used for measuring the positioning errors in machine tools. Ziegert and Mize developed a method to map the volumetric positioning error by using the laser ball bar (LBB) consisting of a laser interferometer aligned with a telescoping ball bar [13]. Castro and Burdekin developed a method to evaluate the positioning accuracy of machine tools and CMM [14]. In recent studies, machine tools have been calibrated by probing the artefact master balls to measure the error parameters during machining.

Erkan and Mayer analyzed the volumetric errors of a five-axe CNC machine tool by probing an uncalibrated artefact [15,2]. Mayer and Erkan also calibrated the 5-axis machine tools by using reconfigurable uncalibrated master balls artifact (RUMBA) [1]. However, there are limited studies on the effect of loading on machine tool errors.

In this project, we aimed to investigate the effect of load on machine geometry with respect to different torques and weights by installing different workpieces on the machine. This allows us estimating the error parameters during machining to compensate them or at least reduce the error values and increase the precision of manufacturing. The compensation can be done by adding the offset manually (in the generated G-code) or automatically (define the offset in the post-processor of the machine tool), fixing the position of the machine axes (such as guideway), etc. Thus, we will be able to recognize the significant machine errors and compensate them before the machining. After compensating the errors, the cutting tool will approach to the exact point of the part which is defined in the G-code file.

To this end, identical heavy blocks (10 Kilograms each) as well as light spacers (less than 1 Kilogram) were designed and fabricated. After assembling the blocks and master balls on the pallet, the machine probed the balls for different indexations from the rotary axes (B and C axes) of the machine using G-codes generated from the RUMBA program. In total, there are 26 types of errors and each error has maximum five coefficients which are called estimated error coefficients. The raw data obtained from machine is then processed to estimate the values of the error coefficients.

There are numerous error sources during machining. Some are systematic and may be compensated. However, the exact number of error parameters is unknown. It is thought that load-induced errors are more significant under certain conditions. In this study, 83 error coefficients which combine axis component errors and axis location errors have been investigated. [16]

The project methodology follows five steps:

1. Design and build means to apply controlled loading conditions;
2. Determine the ball location for the RUMBA method and write appropriate G-code;
3. Conduct the test;
4. Analyze the error to identify possible patterns;
5. Represent each error as a function of loading

### **1.1 Design and build means to apply controlled loading conditions:**

Loading is quantified as force (weight) and related torque. A judicious use of weight units and stacking arrangements allow a sufficient variety of loading conditions for the subsequent analysis. A design based on a “standard” loading unit, a block, was selected. Each block is a ten kilogram steel piece.

When stacking the blocks, it is essential to verify that the combined deflection and tilt of the stack does not create excessive deviations in the master ball positions relative to the machine table. Thus, we have to calculate the maximum deviation and the angle of deviation in the assembled blocks. The stack is modelled as an end-fixed beam and the weight force is a distributed force. The maximum deflection occurs at the end of the beam.

To achieve various loading conditions, some spacers are inserted between the blocks and the table. The spacer is an aluminum piece with a cubic shape. The spacer must be as light as possible to minimize the effect of its weight since the objective of the spacer is to increase torque not weight.

The blocks have been designed and fabricated. The main block is made of steel and is machined. There are four islands at the corners of a block to have a more definite contact with each other and avoid rocking. There are four holes for fixing the blocks together and installing the master balls on them. The spacers also have four holes but have smaller dimensions (due to material stock availability).

## **1.2 Determine the ball location for the RUMBA method and write appropriate G-code**

RUMBA method allows using different master balls in different location of the pallet (Appendix I) [2]. Thus, the number of master balls and their location are adjustable and giving us an opportunity to have more access to different spheres in different combinations. There are some ways for calibrating the artifact master ball:

1. The ball center coordinate is measured as a nominal value. The nominal coordinates for other measurement locations are calculated. The difference between the measured and the calculated coordinates identifies the volumetric errors [17].
2. The artefact is calibrated on a coordinate measuring machine (CMM) and then probe on the machine tools. The difference between these two data can generate the error vectors [18].
3. A 3D artefact is created by mounting a 2D ball plate in different locations. The relation between machine errors and the uncertainty is used to predict the volumetric errors [19],[20].

Having the coordinates of each ball, the G-codes are generated to define the probe trajectory throughout the test. For each test, to probe the balls in different positions, the pallet needs to be rotated in C-axis by 360 degrees and in B-axis by 180 degrees (The B axis will be moved over its range of -90 to +90).

## **1.3 Conduct the test**

All the tests are performed on HU40T 5-axis CNC machine tools. The experimental test is based on the probing the spheres in different combinations (the pallet rotates around its B rotary axes). The probe approaches to the spheres according to the G-code which is generated by RUMBA using Matlab program. The following limitations should be considered:

1. The available blocks:  
12 blocks and 4 spacers have been fabricated with respect to the range of Z axis in the HU40T five axis CNC machine tools.



2. The HU40T standard specification:

All twelve tests have been designed regarding to the machine tools limitation. The maximum possible weight that can be installed on the pallet is 100 kilograms. The maximum distances for Z axis and X axis are 300 mm and 600 mm respectively.

#### **1.4 Analyze the error to identify possible patterns**

After experimental tests, we are able to process the data and estimate the error parameters. With the processed data, it is possible to plot the polynomial graphs of each type of motion error for different loading conditions (different tests). In this section, the goal is to find a possible pattern between load and error values.

#### **1.5 Represent each error as a function of loading**

In this chapter, the objective is to find the correlation matrix to show the level of dependency between the error coefficients and weight and torque. The most correlated error coefficients are identified. Then, each error can be defined as a function of load, torque and both of them simultaneously. Thus, the value of each error coefficient can be estimated for any weight and torque. Finally, residual analysis is done to verify the accuracy of the modeling.

## Chapter 2 Loading fixtures

To investigate the effect of load on the machine geometry, a loading system needs to be mounted on the pallet.

### 2.1 Design stack

The weight of different parts during machining creates a torque relative to the center of the pallet. The loading system consists of a set of standard blocks and standard spacers. This helps us to simultaneously monitor the trend of changes in machine errors with torque and weight by running the test with different weights and different torques. The block is a 10-kilogram steel piece. Figure 2-1 shows the designed block. The block has four holes for being fixed on the pallet or on the other blocks and 4 islands for fixing it on the other blocks and reduce rocking. Some of the blocks have extra hole at the center of the other holes that can be used for placing a master ball on the block.

For achieving more possible loading conditions, a spacer has been designed. The spacer is made of aluminum with four holes (for being fixed on the main blocks). The spacer has been hollowed out as much as possible to reduce the effect of the weight and creates maximum torque. Thus, we can do various tests with different weights and torques. Figure 2-2 shows the designed spacer. Figures 2-3 and 2-4 show the fully dimensioned block and spacer, respectively.

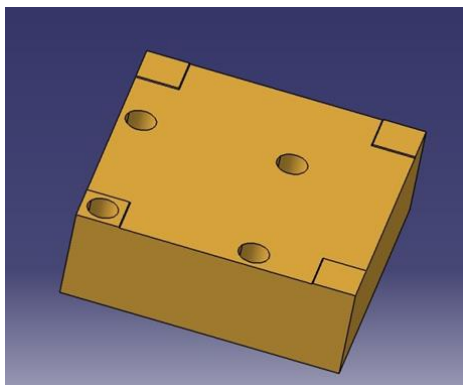


Figure 2-1: Block

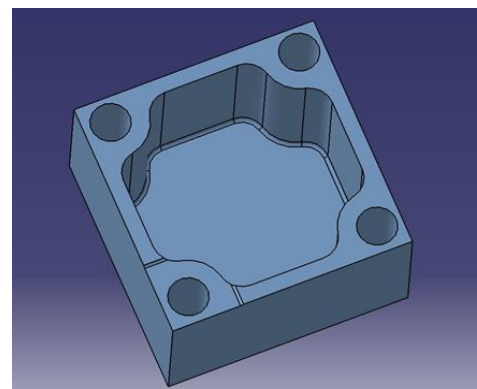


Figure 2-2: Spacer

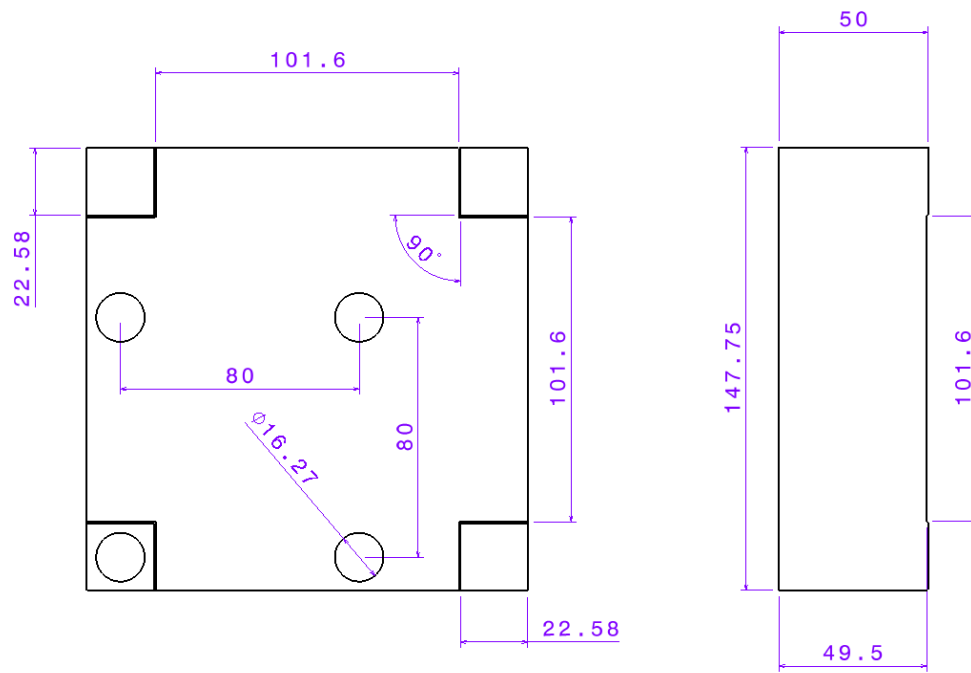


Figure 2-3: The fully dimensioned block

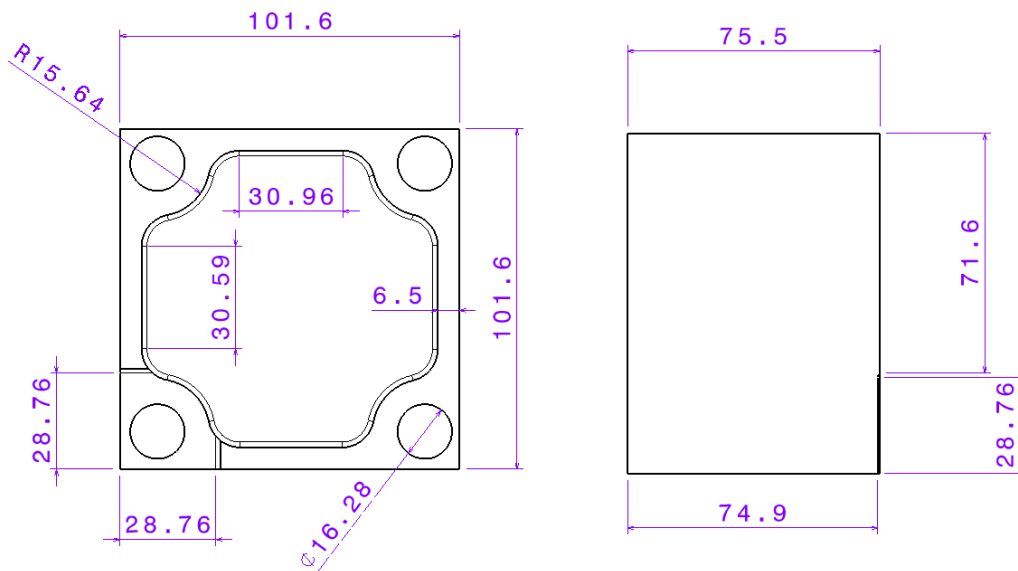


Figure 2-4: The fully dimensioned spacer

## 2.2 Assemble the blocks

The blocks have been assembled symmetrically to avoid creating torque around C-axis of the pallet. The possible patterns for assembling the main blocks and the spacers can be:

- Diagonal: The blocks are installed on a diagonal of the pallet.
- Four corners: In this pattern the combination of blocks and spacers is placed at the corners of the pallet.

Figure 2-5 shows all the possible loading configurations with respect to the designed blocks and the limitations of the machine tools.

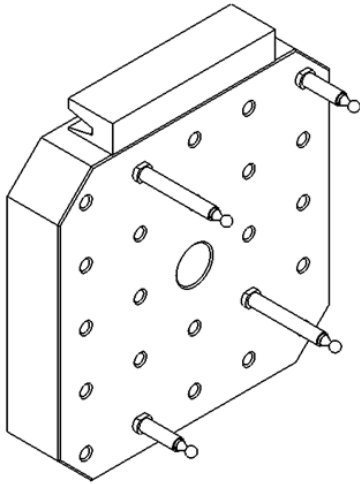


Figure 2-5-1 test 1

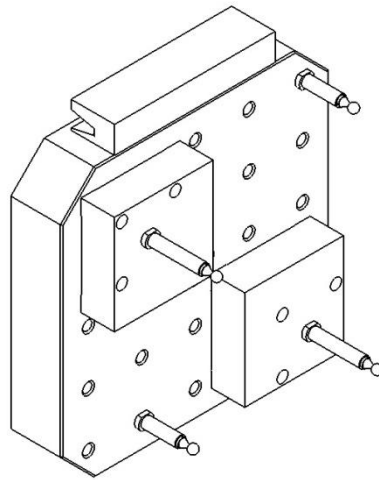


Figure 2-5-2 test 2

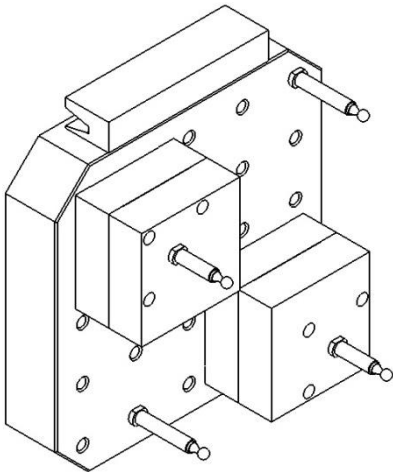


Figure 2-5-3 test 3

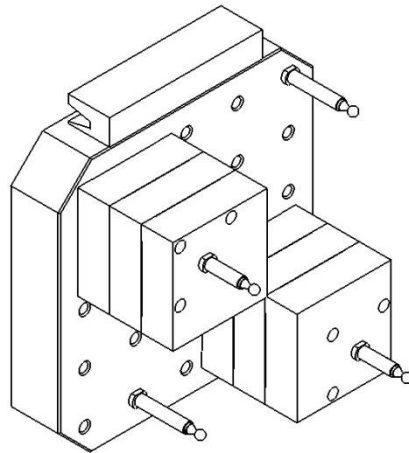


Figure 2-5-4 test 4

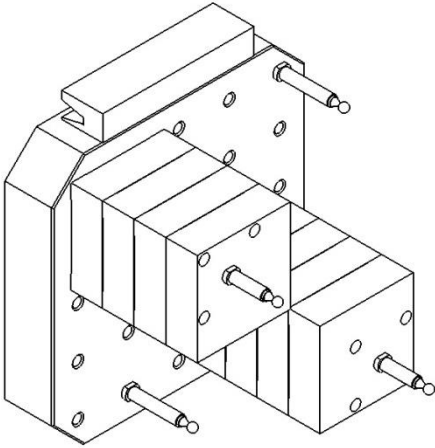


Figure 2-5-5 test 5

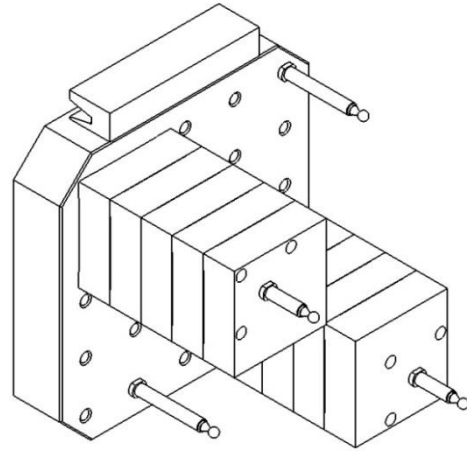


Figure 2-5-6 test 6

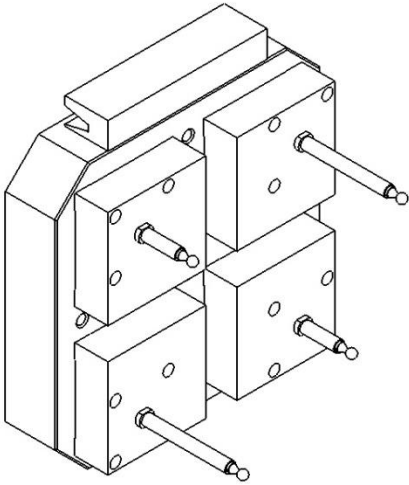


Figure 2-5-7 test 7

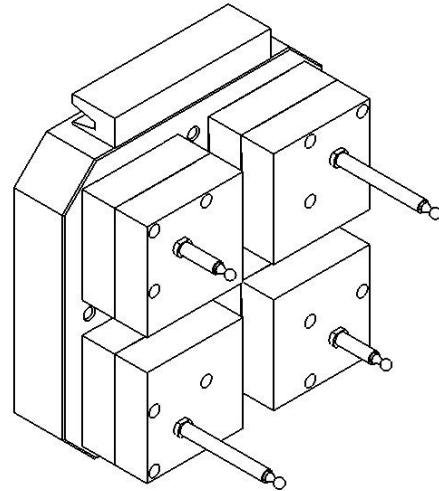


Figure 2-5-8 test 8

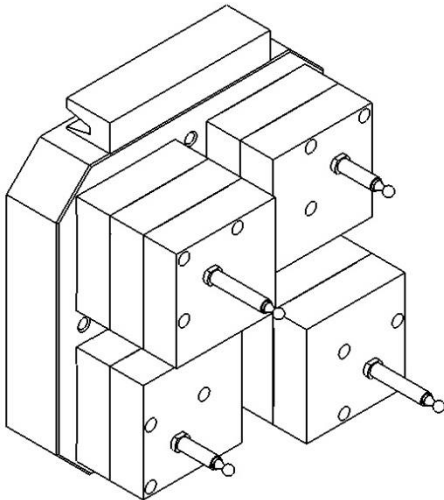


Figure 2-5-9 test 9

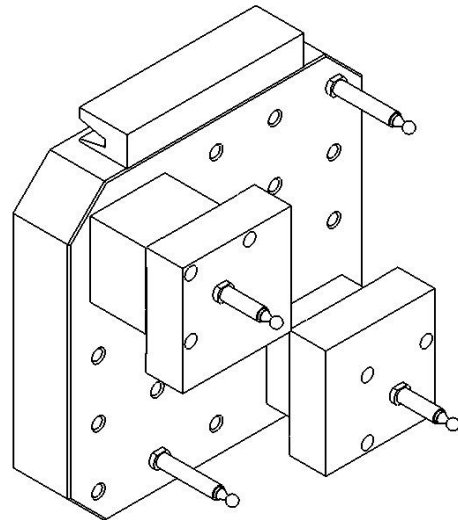


Figure 2-5-10 test 10

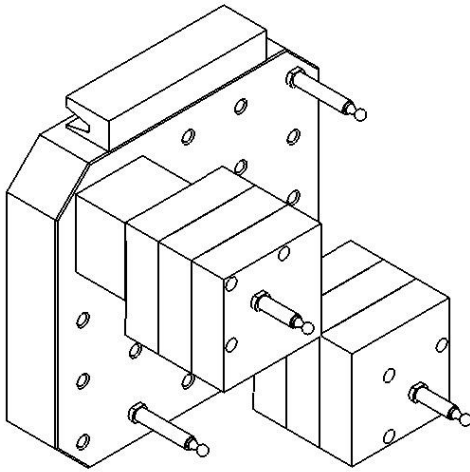


Figure 2-5-11 test 11

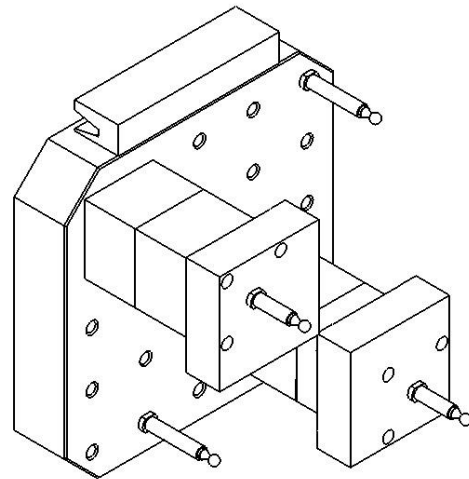


Figure 2-5-12 test 12

Figure 2-5: Different assembly of tests

Table 2-1 presents all the possible tests with the number of blocks and spacers and their patterns. The maximum weight that is permitted to be used on the HU40T CNC machine tools is 100 kilograms.

Table 2-1: The tests specifications

Tests	Pattern	Number of blocks	Weight(N)	Torque(N.m)
1	No pattern	0	0	0
2	Diagonal	2	200	5
3	Diagonal	4	400	20
4	Diagonal	6	600	45
5	Diagonal	8	800	80
6	Diagonal	10	1000	125
7	4 corners	4	400	10
8	4 corners	8	800	40
9	4 corners	10	1000	75
10	Diagonal(with spacer)	2	200	20.1
11	Diagonal(with spacer)	6	600	90.3
12	Diagonal(with spacer)	2	200	35.2

### 2.3 Stack deflection

It is essential to verify that the maximum deflection of a stack of blocks and also the angle of their deflection do not create excessive deviations in the master ball positions, which are placed on the blocks, relative to the machine pallet. Equations 2-1 and 2-2 calculate the maximum deflection and angle of the deflection respectively:

$$y = \frac{w \times L^4}{8 \times E \times I} \quad (2-1) \quad , \quad \Theta = \frac{w \times L^3}{2 \times E \times I} \quad (2-2)$$

$y$ : maximum deflection (m) and  $\Theta$ : angle of deflection (rad). Where  $w$  is weight force (N/m),  $L$  is the total length of whole blocks (m),  $E$  is module of elasticity (= 200 GPa), and  $I$  is area moment of inertia ( $m^4$ ).

$$\text{With } I = \frac{bh^3}{12} = \frac{5.817 \times 5.817^3}{12} = 95.415 \text{ in}^4 = 3.97 \times 10^{-5} \text{ m}^4$$

‘ $b$ ’ and ‘ $h$ ’ are the width and height of the blocks, respectively.

Table 2-2: The values of maximum deflection and the angle of the deflection for all tests

Test	Maximum deflection (y) ( $\mu\text{m}$ )	Angle of deflection ( $\Theta$ ) ( $\mu\text{rad}$ )
1	0	0
2	0.0003	0.0241
3	0.0048	0.1926
4	0.0244	0.6500
5	0.0770	0.1540
6	0.1881	3.0090
7	0.0006	0.0481
8	0.0096	0.3851
9	0.0406	1.0830
10	0.0047	0.1504
11	0.0823	1.4620
12	0.0193	0.3851

As shown in table 2-2, the maximum deflection was for tests with huge load and huge torque. However, the maximum values for the deflection and the angle of the deflection in theory are  $0.1881\text{ }\mu\text{m}$  and  $3.0090\text{ }\mu\text{rad}$ , respectively. Thus, the tests did not create excessive deviations. With above values, it is possible to sketch the graph and show the trend of maximum deflection and the angle of deflection. Figure 2-6 and figure 2-7 are histograms of the linear and angular deflection.

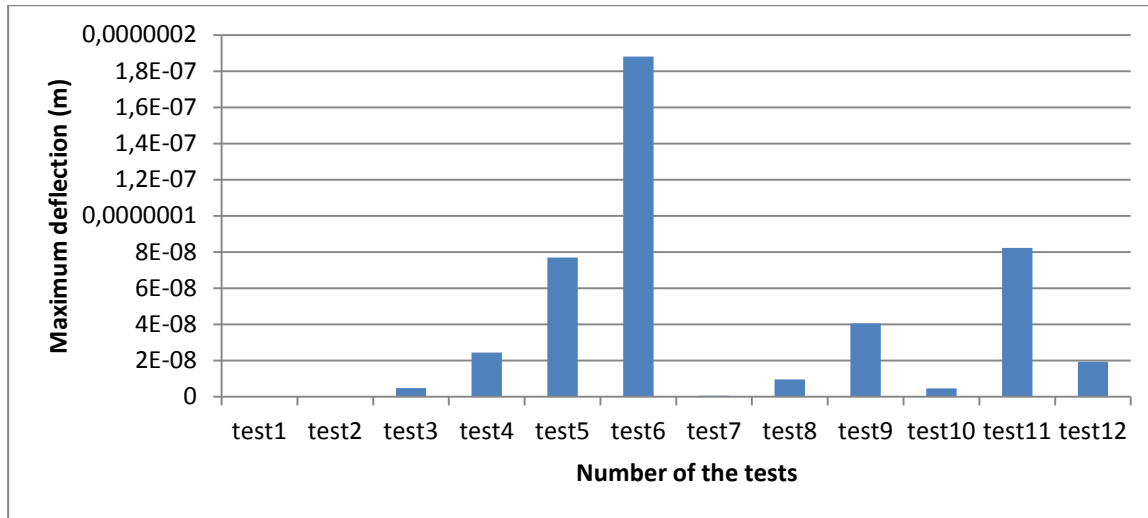


Figure 2-6: The maximum deflection of the blocks for each test

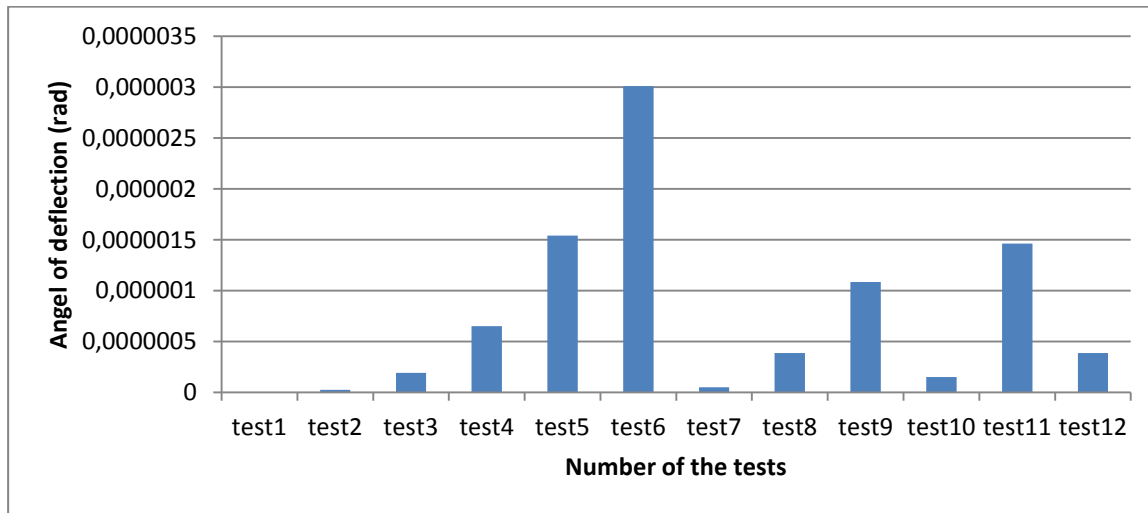


Figure 2-7: The angle of the deflection for each test



### Chapter 3 RUMBA ball location and G-code

RUMBA method uses a combination of master balls on the machine pallet to probe the data [2]. In this method, the number of master balls and their location on the pallet are adjustable. This flexibility helps to probe the master balls for different rotary axes indexations.

Four master balls are used in each test with the same location in X-Y plane to maximize consistency between the tests. We tried to keep the test conditions constant to achieve more accurate results. The length of each master ball can be changed in each test (to facilitate their access for probing). In some tests (with four corners pattern), it is necessary to slightly change the ball location.

The artefact combines sphere, ball plate and magnetic link. The artefacts have been calibrated by calibrating the ball plate and the magnetic ball link separately on CMM. Figure 3-1 shows an artefact.



Figure 3-1: Master ball artefact

In RUMBA method, the master balls do not need to be calibrated with the CMM. Thus, they were placed on the pallet without any calibration.

The probe was mounted in the tool holder (cutting tool) of the machine tool. Figure 3-2 shows the probe, made of MP 700 by Renishaw. The machine probed each ball to determine its exact coordinate from five touches, four at the equator and one at the pole. The points on the equator

were along  $\pm 45^\circ$  diameter. This increases the accessibility of touch points in a variety of artefact locations.

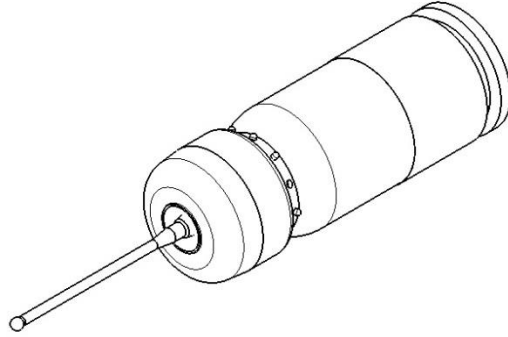


Figure 3-2: Probe

The measurements can vary in different runs and can be influenced by other factors such as thermal effect.

- Repeatability: to verify the repeatability of machine tools, each test was done at least 3 times.
- Thermal effect: The temperature of the machine tools may change during the test. This can affect the accuracy of the results. To minimize the thermal effect and to be consistent, all the tests which last two and half hours were done in the morning when machine was cold.

### 3.1 The mathematical model

The mathematical model has been designed for the HU40T five-axis CNC machine tools. This model represents the nominal modeling, exact modeling and the approximate modeling of the 5-axis CNC machine tools to estimate the values of the error parameters. Before presenting the model, the mathematical model of a joint motion and a link will be calculated.

There are a lot of error sources in machine tools such as geometric deviation, thermal effects, elastic deformation, dynamic effects, NC related errors [16]. The following list defines each type of error:

- Geometric deviation exists in guideways, links and structural elements which can be compensated by physical modification of the machine tools.
- Thermal effects which are caused by heat originating from machine motor, coolant system and the environment. Thermal errors can be classified as quasi-static errors which change slowly in time and are related to the machine structures [21]. This type of error can be reduced by using the machine in a known time and by the maintenance of the motor, guiding system and coolant system.
- Elastic deformations in workpiece or even machine elements such as cutting tool, weight and guideway because of forces.
- Dynamic effects because of inertial forces.
- NC related errors can happen because of the machine tools internal interpolation. NC command is the main part of each machine tool where all the G-code programs are translated to the machine language there.

In this work, only the geometric deviations are considered:

- Mathematical modeling of a joint motion
- Mathematical modeling of a link

### 3.1.1 Mathematical modeling of a joint motion

Figure 3-3 shows the nominal X axis ( $X$ ) and its polynomial curve (actual trajectory  $X'$ ). In this figure, the linear error (black vector) is  $(\overrightarrow{\delta_x})$  and the angular error (red vectors) is  $(\overrightarrow{\varepsilon_x})$ .

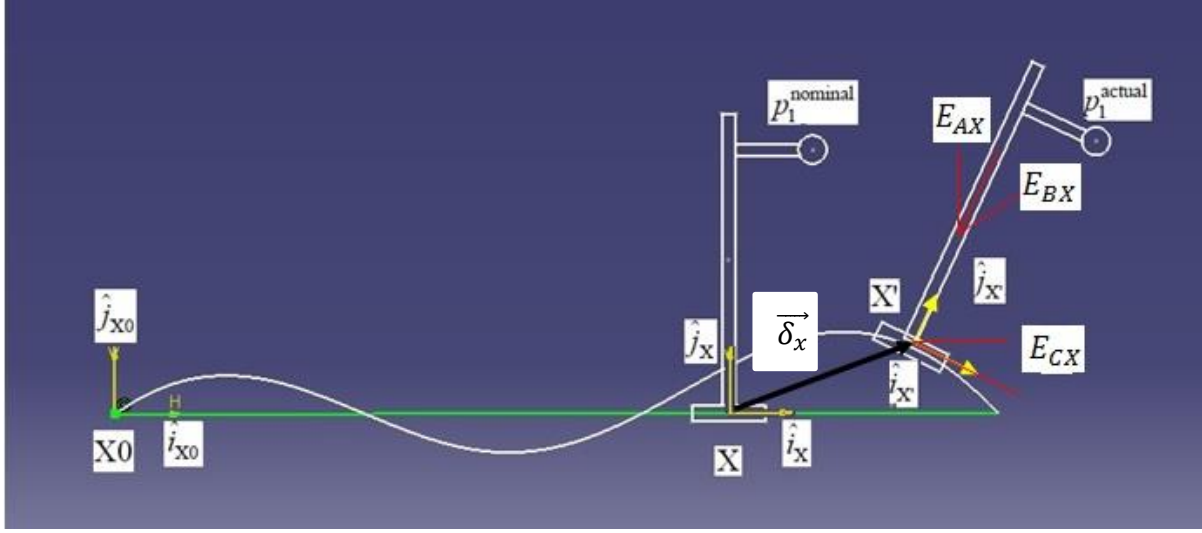


Figure 3-3: Mathematical modeling of a joint motion (black vector is the linear error ( $\vec{\delta}_x$ ) and the red vector is the angular vector ( $\vec{\varepsilon}_x$ ))

Equation 3-1 indicates the relation between the actual coordinate of  $P_1$  and the nominal coordinate in machine frame and X frame respectively.

$${}^{0\}P_1 = \vec{x} + {}^X R(\vec{\delta}_x + {}^X R\{\vec{\varepsilon}_x\}P_1) \quad (3-1)$$

( $\vec{\delta}_x$ ) is the linear error matrix and (R) is the mobile frame rotation. ( $\vec{x}$ ) is the distance between the reference point (zero point) and the nominal joint. (R) in the above equation is equal to:

$${}^0 R = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3-2)$$

$${}^X R = \text{rot}(ECX, \hat{K}) \text{rot}(EBX, \hat{J}) \text{rot}(EAX, \hat{I}) = \text{rot}(\gamma, z) \text{rot}(\beta, y) \text{rot}(\alpha, x) \quad (3-3)$$

$$\text{rot}(ECX, \hat{K}) = \text{rot}(\gamma, z) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$\text{rot}(EBX, \hat{J}) = \text{rot}(\beta, y) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix};$$

$$rot(EAX, \hat{i}) = rot(\alpha, x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix};$$

Equation 3-1 is a vector form of a joint motion modeling. The mathematical model of a joint motion can be also written in homogenous transformation matrix form (HTM). Equation 3-4 represents the HTM model of a joint motion.

$$\begin{bmatrix} {}^{0}\{0\}P_{1,x} \\ {}^{0}\{0\}P_{1,y} \\ {}^{0}\{0\}P_{1,z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} rot(ECX, \hat{k}) rot(EBX, \hat{j}) rot(EAX, \hat{i}) \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} E_{XX} \\ E_{YX} \\ E_{ZX} \\ 1 \end{bmatrix} \begin{bmatrix} {}^{\{x\}}\{x\}P_{1,x} \\ {}^{\{x\}}\{x\}P_{1,y} \\ {}^{\{x\}}\{x\}P_{1,z} \\ 1 \end{bmatrix} \quad (3-4)$$

### 3.1.2 Mathematical modeling of a link

Figure 3-4 shows the model of one link with respect to its nominal and actual position. In this figure, the blue vector ( $\vec{\delta_v}$ ) is a link error and the black vectors ( $\vec{\varepsilon_v}$ ) are the axis location errors.

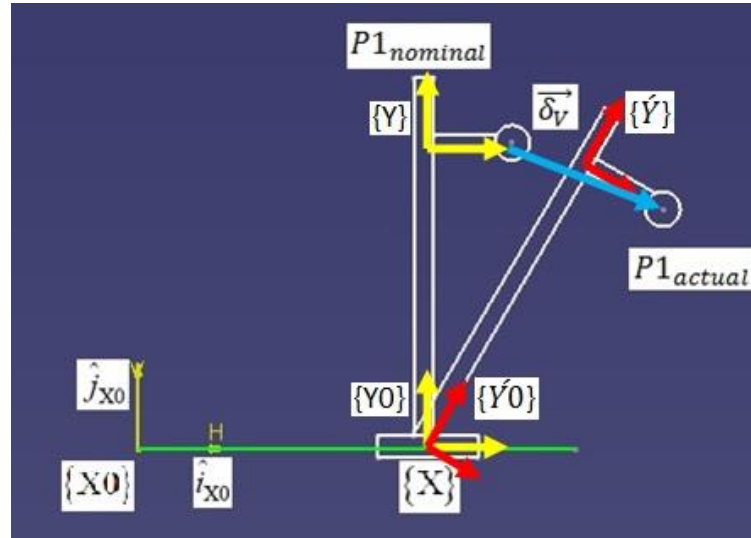


Figure 3-4\_a: Modeling of the link and its parameters

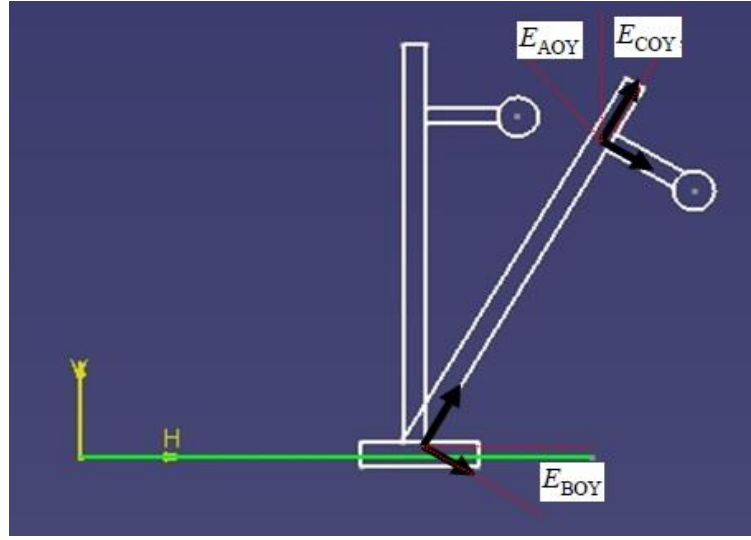


Figure 3-4\_b: Modeling of the link and its axis location errors

The vector form of the link model is presented in equation 3-5:

$$\{x0\},x0P_1 = \vec{x} + \begin{bmatrix} XOY \\ YOY \\ ZOY \end{bmatrix} + {}^{x0}_Y R \left( \begin{bmatrix} E_{XOY} \\ E_{YOY} \\ E_{ZOY} \end{bmatrix} + {}^{y0}_Y R (\vec{y} + \{Y\},Y P_1) \right) \quad (3-5)$$

Equation 3-5 can be rewritten as equation (3-6):

$$\begin{aligned} \{x0\},x0P_1 = & \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} XOY \\ YOY \\ ZOY \end{bmatrix} + (rot (COY, \hat{k}) rot (BOY, \hat{j}) rot (AOY, \hat{i})) \left( \begin{bmatrix} E_{XOY} \\ E_{YOY} \\ E_{ZOY} \end{bmatrix} + rot(E_{COY}, \hat{k}) \right. \\ & \left. rot(E_{BOY}, \hat{j}) rot(E_{AOY}, \hat{i}) \left( \begin{bmatrix} y \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \{Y\},Y P_{1,x} \\ \{Y\},Y P_{1,y} \\ \{Y\},Y P_{1,z} \end{bmatrix} \right) \right) \end{aligned} \quad (3-6)$$

Where  $\{x0\},x0P_1$  is the numerical component of vector  $x0P_1$  projected in frame  $\{X0\}$ . ' $E_{XOX}$ ', ' $E_{YOX}$ ', ' $E_{ZOX}$ ' are X offset of X, Y offset of X, Z offset of X respectively. ' $E_{COY}$ ', ' $E_{BOY}$ ', ' $E_{AOY}$ ' are axis location errors of the Y-axis around the z, y and x axis respectively. The HTM for this model can be presented in equation (3-7):

$$\{x0\},x0P_1 = {}^{x0}_X T {}^{x0}_Y T {}^{y0}_Y T {}^{y0}_Y T \{Y\},Y P_1 \quad (3-7)$$

which can be expanded by equation (3-8):

$$\begin{bmatrix} {}^{X0},X0P_{1,x} \\ {}^{X0},X0P_{1,y} \\ {}^{X0},X0P_{1,z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \text{rot}(COY, \hat{k}) \text{rot}(BOY, \hat{j}) \text{rot}(AOY, \hat{i}) & XOY \\ & YOY \\ & ZOY \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{rot}(E_{COY}, \hat{k}) \text{rot}(E_{BOY}, \hat{j}) \text{rot}(E_{AOY}, \hat{i}) & E_{XOY} \\ & E_{YOY} \\ & E_{ZOY} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{Y},YP_{1,x} \\ {}^{Y},YP_{1,y} \\ {}^{Y},YP_{1,z} \\ 1 \end{bmatrix} \quad (3-8)$$

The whole 5-axis machine tool is modelled by successively combining the two cited models of the joint motion and the link of the machine. The model of the cutting tool (spindle) and the workpiece have to also be considered in the modeling.

### 3.1.3 Nominal modeling

The HU40T five-axis CNC machine tool has a topology WCBXFZYST, where ‘W’, ‘F’, ‘S’, and ‘T’ are for workpiece, foundation, spindle and tool frame respectively. ‘C’ and ‘B’ represent the rotary axes of the machine; ‘X’, ‘Y’ and ‘Z’ represent the prismatic axes of the machine. The nominal modeling of the machine presents the machine chain kinematic regardless to the errors of the machine. Thus, in the nominal machine, it is assumed that the machine works perfectly accurate. Figure 3-5 shows the nominal kinematic chain.

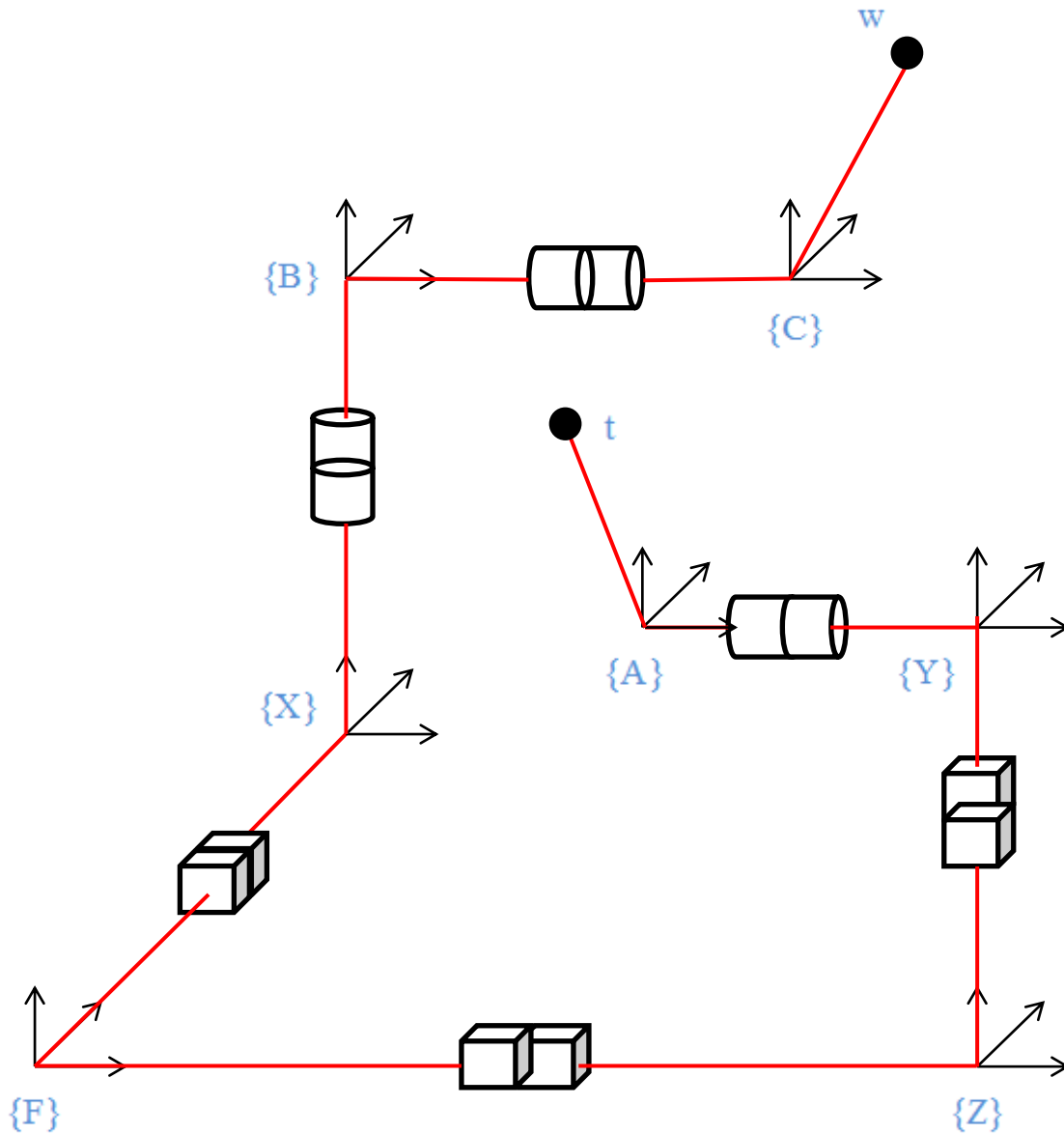


Figure 3-5: The nominal kinematic chain of the 5-axis CNC machine tool

### 3.1.4 Exact modelling

In the exact modelling, all the error parameters are considered. There are six errors (three linear and three angular) for each prismatic axes and there are also six errors (three radial motion, two tilt error motion and one indexation error angular) for each rotary axis of the machine tools. Figure 3-6 shows the kinematic chain of the exact modelling of the machine.



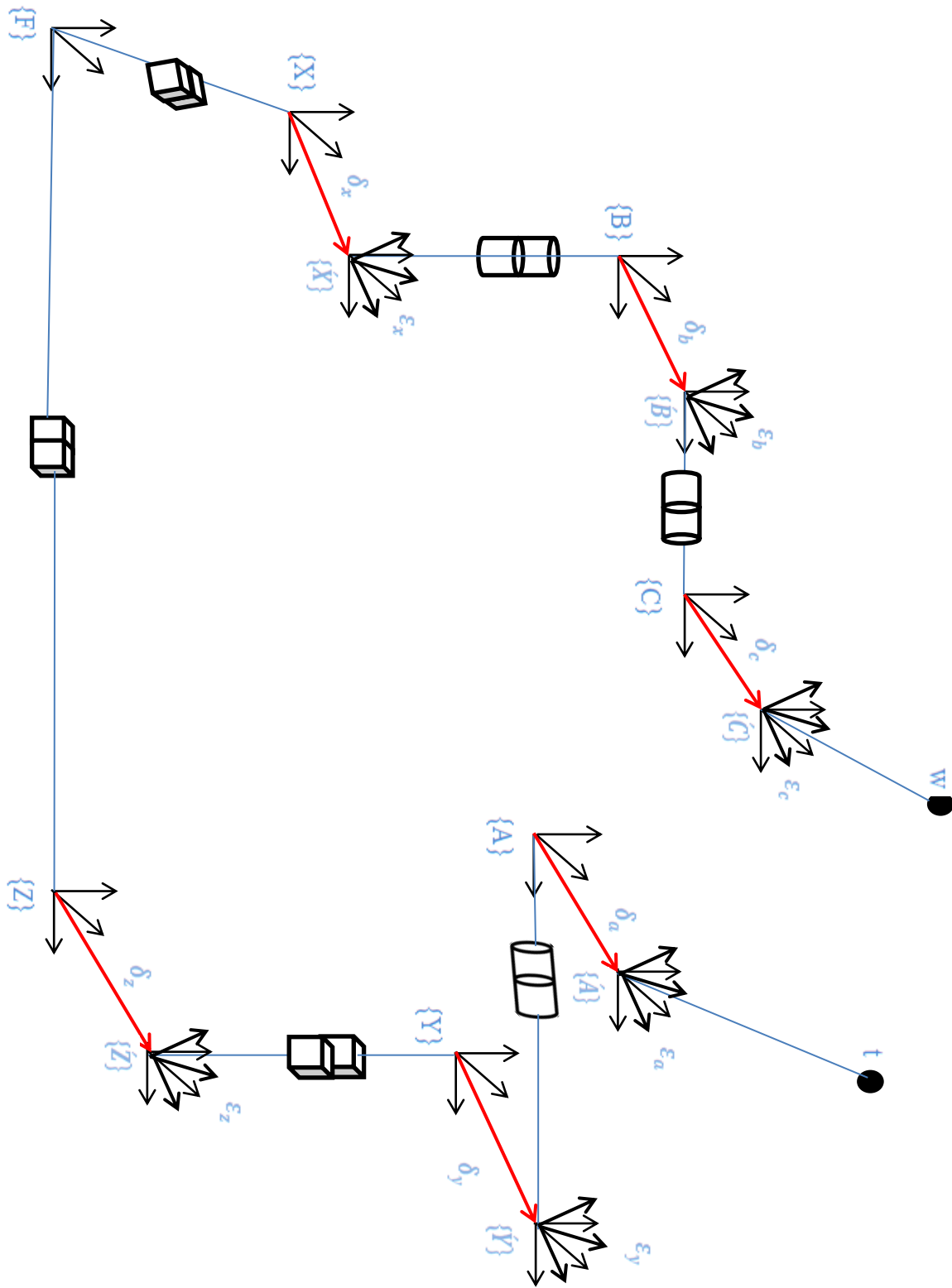


Figure 3-6: The exact kinematic chain of the 5-axis CNC machine tool

Below list presents all the motion errors from the exact modeling which can be occurred in the machine tools:

1. 'EXX' : Linear positioning error of X axis
2. 'EYX' : Straightness error of X axis in Y direction
3. 'EZX' : Straightness error of X axis in Z direction
4. 'EAX' : Angular error of X axis around X axis (roll)
5. 'EBX' : Angular error of X axis around Y axis (pitch)
6. 'ECX' : Angular error of X axis around Z axis (yaw)
7. 'EXY' : Straightness error of Y axis in X direction
8. 'EYY' : Linear positioning error of Y axis
9. 'EZY' : Straightness error of Y axis in Z direction
10. 'EAY' : Angular error of Y axis around X axis
11. 'EBY' : Angular error of Y axis around Y axis
12. 'ECY' : Angular error of Y axis around Z axis
13. 'EXZ' : Straightness error of Z axis in X direction
14. 'EYZ' : Straightness error of Z axis in Y direction
15. 'EZZ' : Linear positioning error of Z axis
16. 'EAZ' : Angular error of Z axis around X axis
17. 'EBZ' : Angular error of Z axis around Y axis
18. 'ECZ' : Angular error of Z axis around Z axis
19. 'EXA' : Axial error motion of A axis in X axis
20. 'EYA' : radial motion error of A axis in Y axis
21. 'EZA' : radial motion error of A axis in Z axis
22. 'EAA' : Angular positioning error motion of A axis around X axis
23. 'EBA' : Tilt error motion of A axis around Y axis
24. 'ECA' : Tilt error motion of A axis around Z axis
25. 'EXB' : radial motion error of B axis in X axis
26. 'EYB' : Axial error motion of B axis in Y axis
27. 'EZB' : radial motion error of B axis in Z axis
28. 'EAB' : Tilt error motion of B axis around X axis
29. 'EBB' : Angular positioning error motion of B axis around Y axis

30. 'ECB' : Tilt error motion of B axis around Z axis
31. 'EXC' : radial motion error of C axis in X axis
32. 'EYC' : radial motion error of C axis in Y axis
33. 'EZC' : Axial error motion of C axis in Z axis
34. 'EAC' : Tilt error motion of C axis around X axis
35. 'EBC' : Tilt error motion of C axis around Y axis
36. 'ECC' : Angular positioning error motion of C axis around Z axis

All the equations for nominal and exact modeling have been presented in appendix III.

### 3.2 Generating G-code

RUMBA method generates G-code after defining the balls locations and the accessible combinations (which ball can be probed and which cannot) [2]. The G-code is written according to the post-processor of the HU40T 5-axis CNC machine tool. In programming G-code, it is necessary to use right commands at the right place. Here, the commands are presented [23]:

- N: The number of the blocks (N11 means the 11<sup>th</sup> block in G-code program)
- G: Functions related to the spindle motions.
  1. G0: Rapid positioning
  2. G01: Linear interpolation
  3. G02, G03: Circular interpolation
  4. G17, G18, G19: Plane selection
  5. G90, G91: Position system
  6. G20, G70, G21, G71: Length units (inch, mm)
  7. G40, G41, G42: Tool Radius Compensation
- F: Feedrate function
- M: Miscellaneous codes
  1. M00: Program stop
  2. M01: Optional stop
  3. M30: Program end
  4. M03: Spindle on and rotate clockwise
  5. M04: Spindle on and rotate counter-clockwise

6. M05: Spindle off to stop
  7. M19: Spindle off to stop at a certain angle
- S: Spindle speed function
  - T: Tool function

The RUMBA method uses the specific numbers to call the coordinates of each sphere in different rotary axes indexation. Here, a part of the G-code is shown:

#15=0.000000	#11=120.000000	The Y location (mm)
#4=0.000000	#12=120.000000	The Z location (mm)
#9=1.000000	M19A#4	
#500=-90.000000	G0 B#500 C#501	
#501=-180.0000	G0 X#10 Y#11	
#10=151.600000		The X location (mm)

We should take note that the coordinate frame changes in each rotary axes indexation which means that the axes have different directions during rotating the pallet. For instance, when the pallet is in  $B=0$  and  $C=0$ , the linear axes are defined as shown in figure 3-7, but when the pallet moves to  $B=-90^\circ$ , the axes will have a new direction same as in figure 3-8.

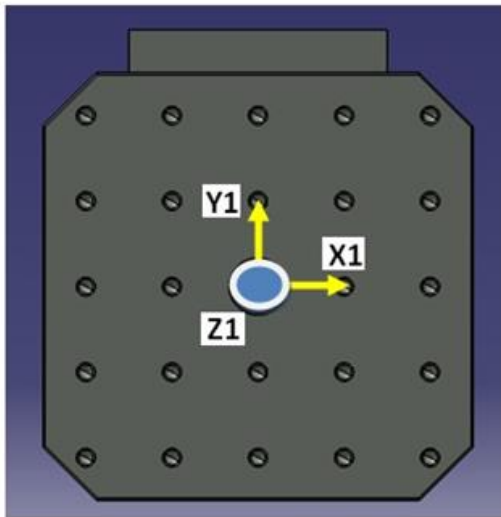


Figure3-7: Machine Pallet in  $B=0^\circ$ ,  $C=0^\circ$

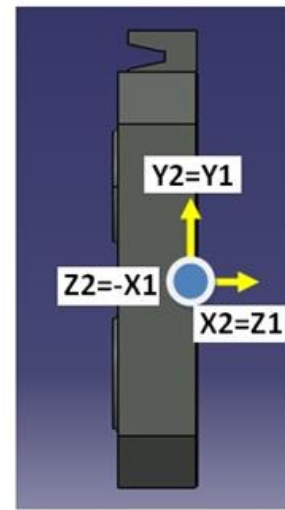


Figure3-8: Machine Pallet in  $B=-90^\circ$ ,  $C=0^\circ$

Figure 3-9 shows a flowchart for generating G-code in RUMBA. We should follow the flowchart to produce a right G-code.

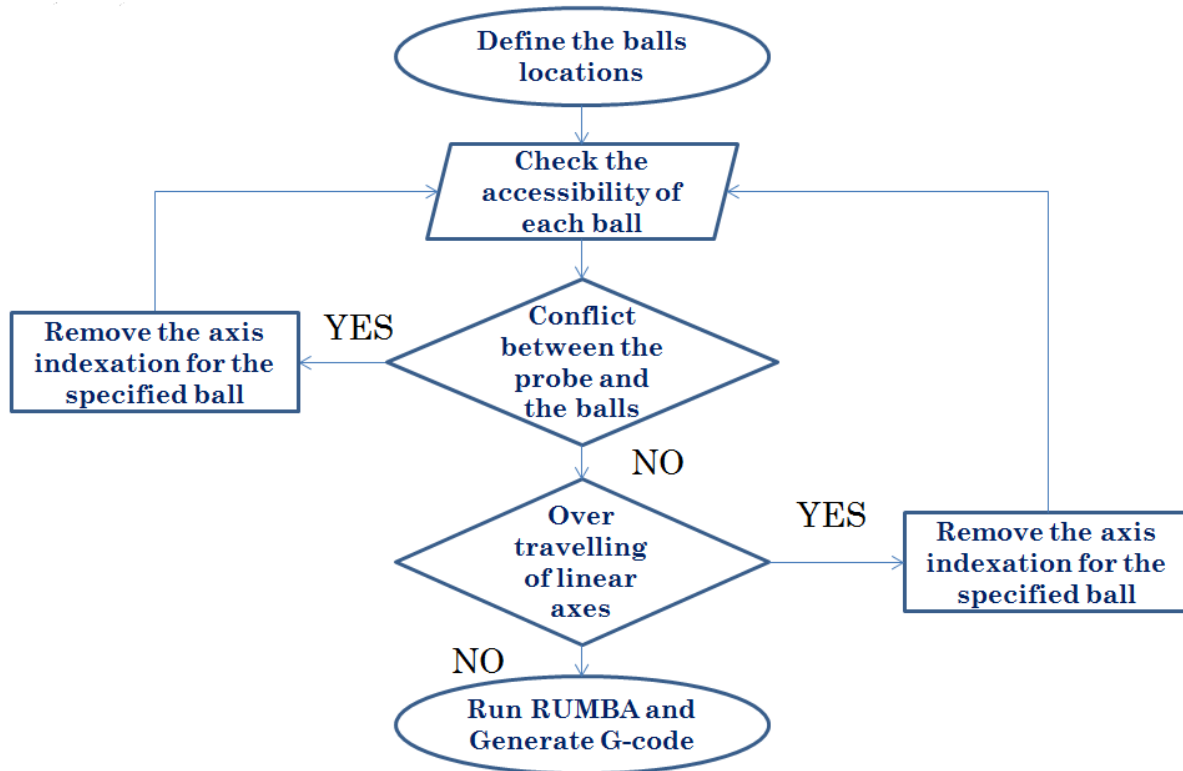


Figure 3-9: The flowchart for generating G-code in RUMBA

## Chapter 4 Conduct the test

### 4.1 Machine tool

Tests were conducted on a 5-axis machine tool in the laboratoire de recherche en fabrication virtuelle (LRFV) at École Polytechnique de Montréal. The machine is a MITSUI SEIKI 5-Axis High Production Machining Center Model HU40T. Table 4-1 presents the standard features of the cited machine.

Table4-1: Standard Features of the machine tool

<b>Working Capacity</b>	
Table longitudinal stroke (X axis)	610 mm (24.02")
Spindle head vertical stroke (Y axis)	560 mm (22.05")
Column travel stroke (Z axis)	560 mm (22.05")
<b>Interference of whole machine cover with work piece on pallet</b>	
In machine	500 mm Ø (19.6" Ø)
In Optional APC	500 mm Ø (19.6" Ø)
<b>Feed Rate</b>	
X, Y and Z axes	36,000 mm/min (1,417 IPM)
B axis	0.001°: 7,200°/min. (20 rpm)
C axis	0.001°: 7,200°/min. (20 rpm)
<b>Cutting feed</b>	
X, Y and Z axes	0.1~20,000 mm/min. (0.004~787 IPM)
B axis	0.001°: 0.1~7,200°/min.
C axis	0.001°: 0.1~7,200°/min.
<b>Minimum Resolution</b>	
X, Y and Z axes	0.001 mm
B & C axis	0.001°
<b>Power Required</b>	
Electrical Service Required	AC 200/220V 3 phase 50/60 HZ 60 KVA

<b>Air required</b>	
Air Service Required	0.5 - 0.7 Mpa, 0.7 m <sup>3</sup> /min. (70 ~ 110 psi @ 25 cfm) (Dry and clean air)
Air Service for 12,000 RPM Spindles and Higher	0.5 - 0.7 Mpa, 1.0 m <sup>3</sup> /min. (70 ~ 110 psi @ 42 cfm) (Dry and clean air)
<b>Machine Dimensions</b>	
Length	5,800 mm (229")
Width	3,930 mm (155")
Height	2,792 mm (110")
Machine Weight	13,500 kg. (29,700 lb.)
	Numerical Control Equipment
CNC Control	FANUC 30i MA

The experimental tests have been designed with respect to the machine's features. For instance, the maximum space for the blocks in Z-direction is 300 mm and each designed block has 50 mm height. Thus, it is impossible to put 7 blocks together on the pallet (the total height will be 350 mm). The other limitation is the maximum loads of 100 kilograms.

The G-code file is transferred to the NC of the machine. The NC reads the commands in the program and translates it to the machine language. Thus, the probe approaches to the exact position of the spheres. Figure 4-1 shows the HU40T 5-axis machine tool.



Figure4-1: MITSUI SEIKI 5-Axis High Production Machining Center Model HU40T

## 4.2 Master balls

Figure 4-2 shows one artefact master ball. These artefacts are used for probing. The artefact combines a sphere and a bar. At the base of the artefact, there is an adapter to screw it into the pallet holes. We can also install the master balls on the designed blocks with a thread changer. The material of the sphere is ceramic. The probe, which will be explained in the next section, touches the master ball five times to calculate the apparent position of the center of the ball.

In this method, it is not necessary to calibrate the artefacts with CMM before probing. We only need to maintain the sphericity and the surface quality of the master balls [25].



Figure 4-2: The artefact master ball



### 4.3 Probe

The probe is a Renishaw MP700 with 360 degree transmission system. Spindle mounted optical transmission measurement probe is used for high accuracy component setting and inspection on medium to large machining centers. Table 4-2 gives the probe components.

Table 4-2: The probe components

MP700 probe	3D touch inspection probe
OMP	optical module probe
OMM	optical module machine
O-M-I	optical machine interface
PSU3	power supply unit for O-M-I or MI 12
Probing software	is available for most types of machine control

The stylus of the probe is adjustable. It means that there are number of styluses with different lengths. Their usage depends on the accessibility for probing the master balls.

In this study, it was preferred to use the long stylus for the tests for more accessibility to the master balls in different combinations of A, B and C axes rotations. As longer stylus can make larger errors, the length of the stylus should be optimized. Figure 4-3 shows the actual probe which was used in this study.



Figure 4-3: The MP 700 Renishaw probe

Before using the probe, it is calibrated with a gildemeister-devlieg system-werkzeuge gmbh microset. This device shows the deviation of the stylus from the reference (zero) point. Figure 4-4 shows this microset which combines four main parts as follows:

1. First monitor: This monitor represents the X.Y and Z coordinates of the tip of stylus. Thus, we can detect the values of stylus deviation in each direction.
2. The hole: The probe is placed in the hole to be fixed on the system.
3. Second monitor: This monitor indicates the geometry of the tip of stylus. Thus, it is possible to compensate the deviations by returning the stylus to zero point.
4. The Column: modifies the height of the probe with respect to the length of stylus.

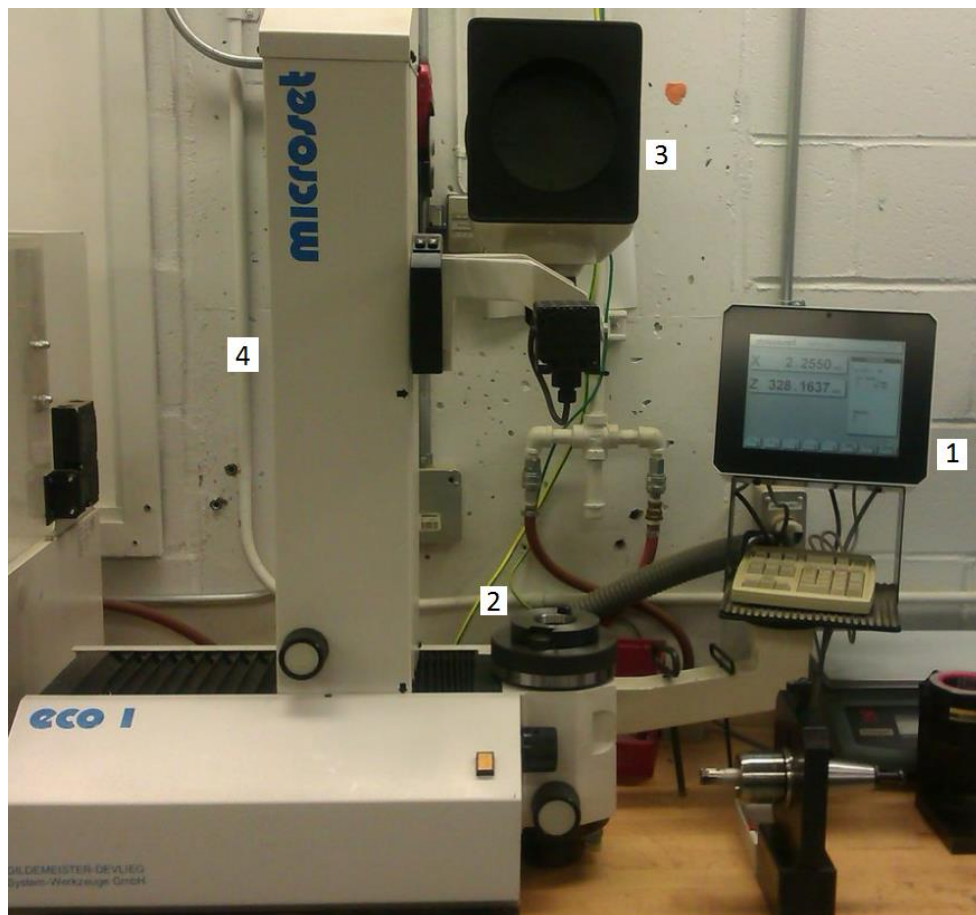


Figure 4-4: Gildemeister-devlieg system-werkzeuge gmbh microset

#### 4.4 Block

The designed block was  $148 \times 148 \times 50$  mm. The material was steel and its weight was equal to 10 kilograms. There were 4 holes on the block which are the place for screws. There were 4 islands at the corners of the block. These islands help fixing blocks on each other more smoothly and also avoid slipping. On the surface of some blocks, there was another hole to install the master ball on them. \the aim was to design the blocks with a simple geometry to install them easily on the pallet. Figure 4-5 shows the designed block.

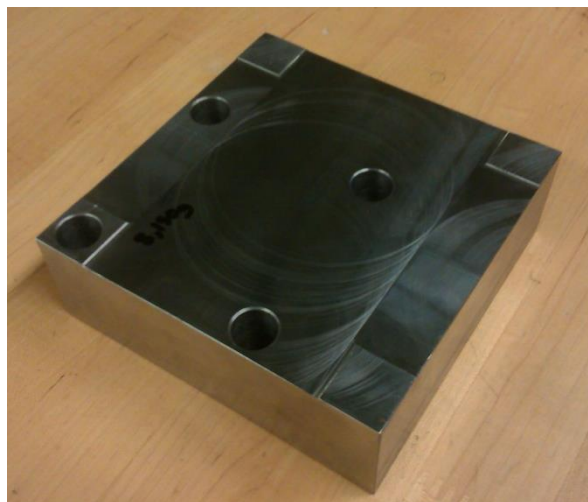


Figure 4-5: The fabricated block

#### 4.5 Spacer

The spacer was designed to keep the heavy steel blocks away from the pallet surface. The spacers helped to reach the maximum allowable machine torque. The spacer was made of aluminum and its dimension was  $102 \times 102 \times 75$  mm. The spacer was hollowed out as much as possible to reduce the effect of the weight and reach the maximum torque. There are 4 holes on the spacer for fixing it on the blocks or on the pallet. The spacers were placed on the pallet as shown in chapter 2. Figure 4-6 illustrates the designed spacer.



Figure 4-6: The fabricated spacer

#### 4.6 Set of axes indexations

In this study, experimental tests have been done with the same set of rotary axes indexations. As mentioned before, it is better to keep all conditions constant in the different tests to have a better comparison. All the axes indexations which were used in this project are shown in figure 4-8. In these figures, there is no loading fixture and the aim is only to show the pallet in different locations. Figure 4-7 shows the position of the B and C axes.

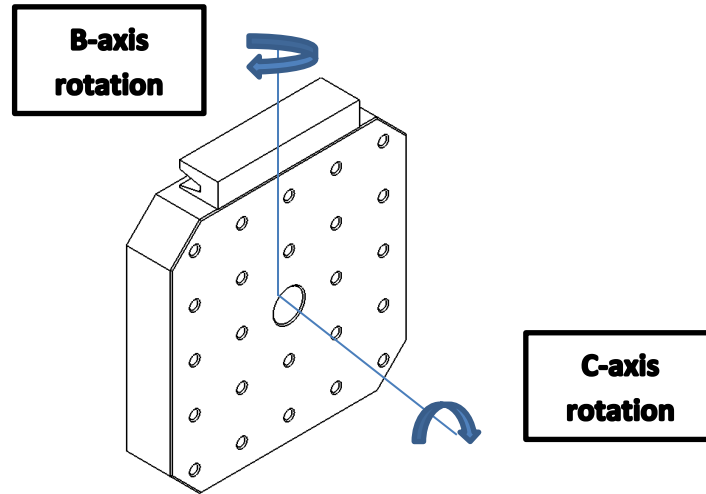
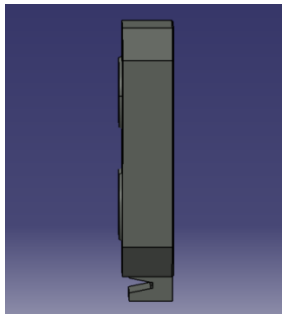
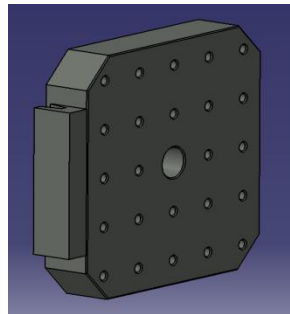


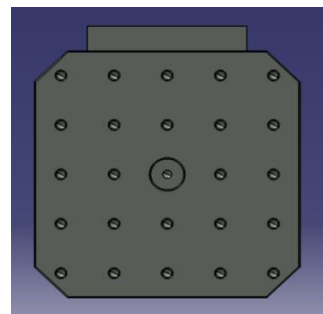
Figure 4-7: The position of B and C rotary axes of the machine pallet



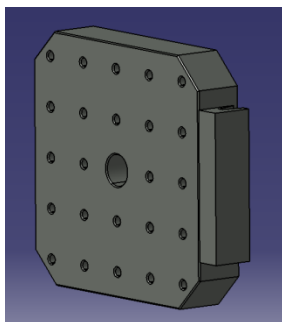
B=-90°, C=-180°



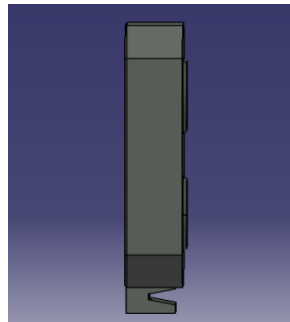
B=-45°, C=-90°



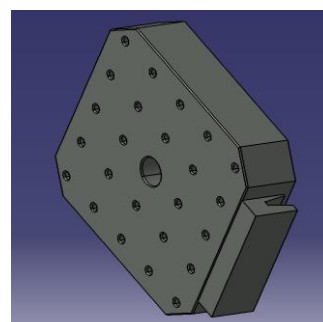
B=0°, C=0°



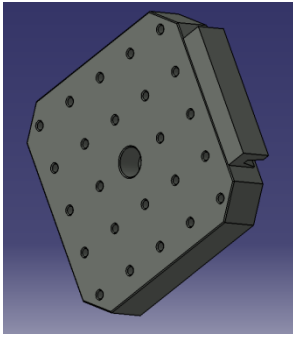
B=45°, C=90°



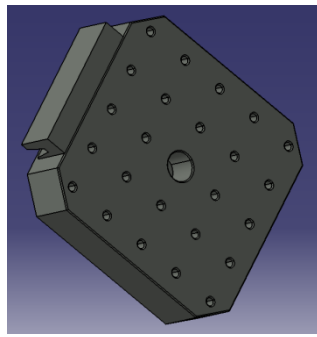
B=90°, C=180°



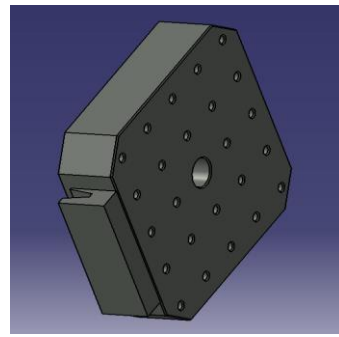
B=60°, C=135°



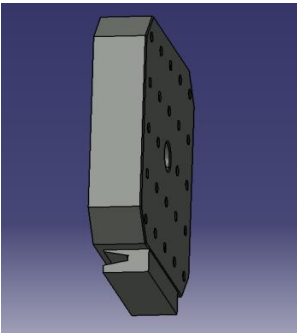
$B=30^\circ, C=45^\circ$



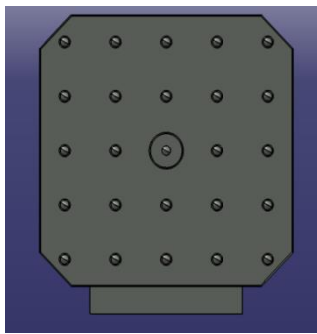
$B=-30^\circ, C=-45^\circ$



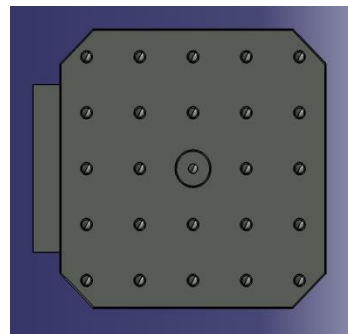
$B=-60^\circ, C=-135^\circ$



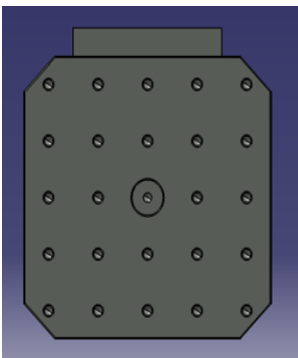
$B=-75^\circ, C=-170^\circ$



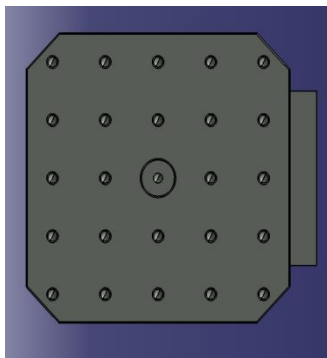
$B=0^\circ, C=-180^\circ$



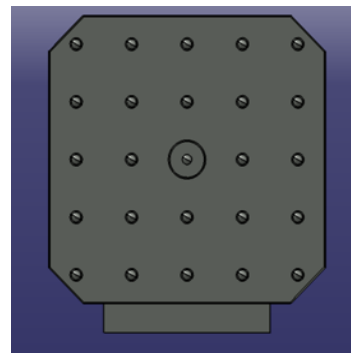
$B=0^\circ, C=-90^\circ$



$B=0^\circ, C=0^\circ$



$B=0^\circ, C=90^\circ$



$B=0^\circ, C=180^\circ$

Figure 4-8: 15 of 19 axes indexations

There are 19 combinations for each test. Figure 4-8 indicates 15 out of 19 combinations. For the last 4 combinations, The B and C axis are set to zero and the A axis (spindle rotation) are set to 0, 90, 180 and 270 degrees, respectively.

Four artefact master balls are set successively on the pallet in diagonal blocks pattern and in four corners blocks pattern. The locations in X and Y axes are constant in diagonal pattern and four corners pattern separately. This stability eliminates the result variation. The locations of master ball are shown in figure 4-9. Table 4-3 also shows the nominal master balls coordinates for all 12 tests (12 weights and torques).

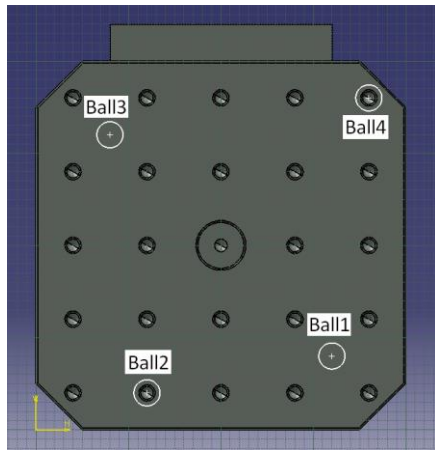


Figure 4-9\_a: Diagonal pattern

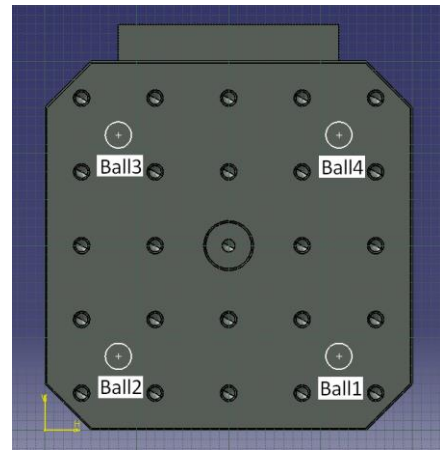


Figure 4-9\_b: 4 Corners pattern

Table 4-3: The nominal master ball coordinates for different tests

The number of the test		X coordinate (mm)	Y coordinate (mm)	Z coordinate (mm)	Bar length(in)
Test1	Ball1	80	-80	127	5
	Ball2	-80	-160	76.2	3
	Ball3	-80	80	152.4	6
	Ball4	160	160	76.2	3
Test2	Ball1	120	-120	151.6	4
	Ball2	-80	-160	76.2	3
	Ball3	-120	120	151.6	4
	Ball4	160	160	76.2	3
Test3	Ball1	120	-120	176.2	3
	Ball2	-80	-160	101.6	4
	Ball3	-120	120	176.2	3

	Ball4	160	160	101.6	4
Test4	Ball1	120	-120	226.2	3
	Ball2	-80	-160	152.4	6
	Ball3	-120	120	251.6	4
	Ball4	160	160	127	5
Test5	Ball1	120	-120	276.2	3
	Ball2	-80	-160	101.6	4
	Ball3	-120	120	276.2	3
	Ball4	160	160	101.6	4
Test6	Ball1	120	-120	326.2	3
	Ball2	-80	-160	177.8	7
	Ball3	-120	120	351.6	3
	Ball4	160	160	127	5
Test7	Ball1	120	-120	126.2	3
	Ball2	-120	-120	177	5
	Ball3	-120	120	126.2	3
	Ball4	120	120	151.6	4
Test8	Ball1	120	-120	176.2	3
	Ball2	-120	-120	227	5
	Ball3	-120	120	176.2	3
	Ball4	120	120	201.6	4
Test9	Ball1	120	-120	226.2	3
	Ball2	-120	-120	227	5
	Ball3	-120	120	226.2	3
	Ball4	120	120	201.6	4
Test10	Ball1	120	-120	201.7	3
	Ball2	-80	-160	127	5
	Ball3	-120	120	201.7	3
	Ball4	160	160	101.6	4
Test11	Ball1	120	-120	301.7	3
	Ball2	-80	-160	177.8	7
	Ball3	-120	120	301.7	3
	Ball4	160	160	152.4	6
Test12	Ball1	120	-120	277.2	3
	Ball2	-80	-160	177.8	7
	Ball3	-120	120	277.2	3
	Ball4	160	160	101.6	4



#### 4.7 Validation test

In chapter two, the maximum deflection and the angel of the deflection for different loading conditions were calculated theoretically. But it is important to validate the results with a practical test to prove that there is no significant deflection on the blocks. Thus, an experimental test was defined for verifying the deflection of the blocks on the machine. In this test, 10 blocks were mounted diagonally on the pallet of the machine. A magnetic ball bar Renishaw was then used to show the distortion of its length. One magnetic sphere was mounted on a side of the top block and the other was placed on the pallet. The Renishaw ball bar was then connected to the computer to show the exact length of the ball bar at each 5 degree of C-axis rotation. The nominal length of the ball bar was equal to 150 mm. The ball bar should be first calibrated and then placed on the machine. Figure 4-10 and figure 4-11 show the set up for validation test. Figure 4-12 presents the length variation of the ball bar for different C-axis rotations.

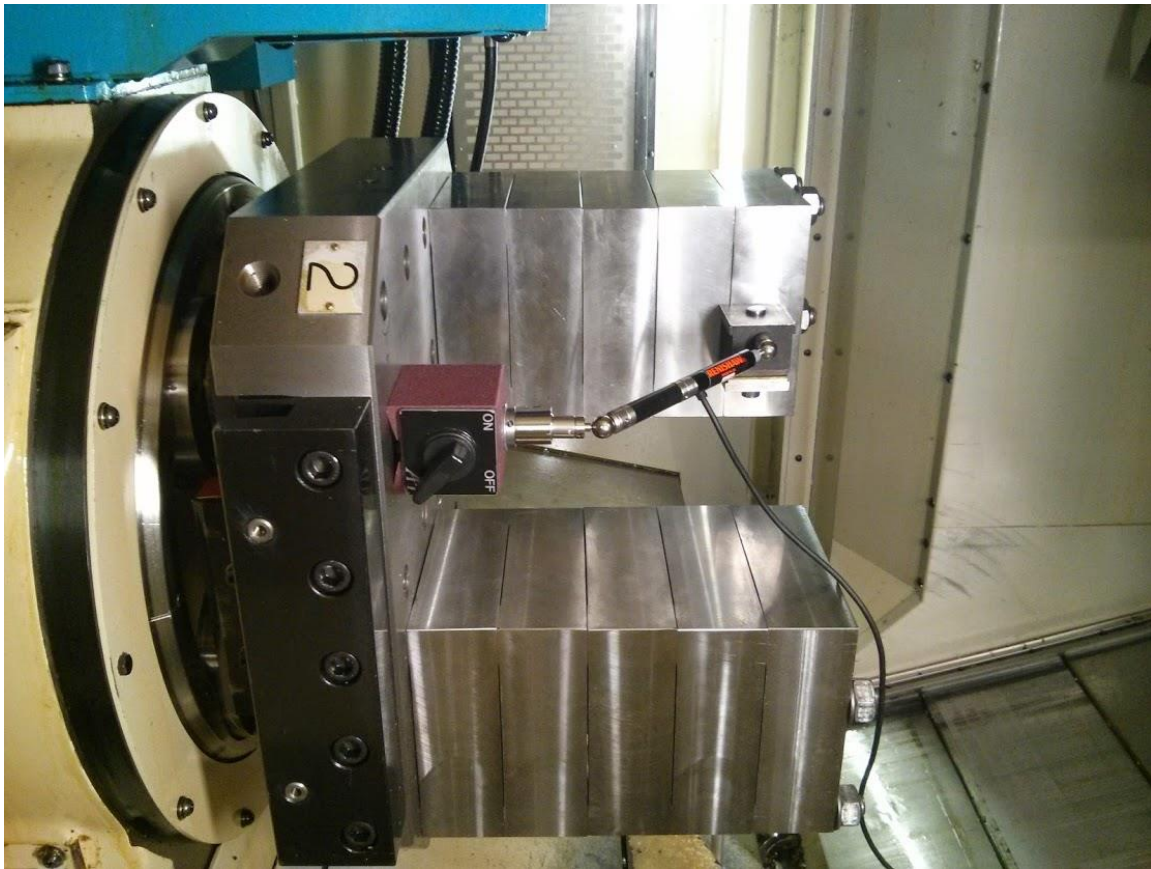


Figure 4-10: The validation test set up with only blocks



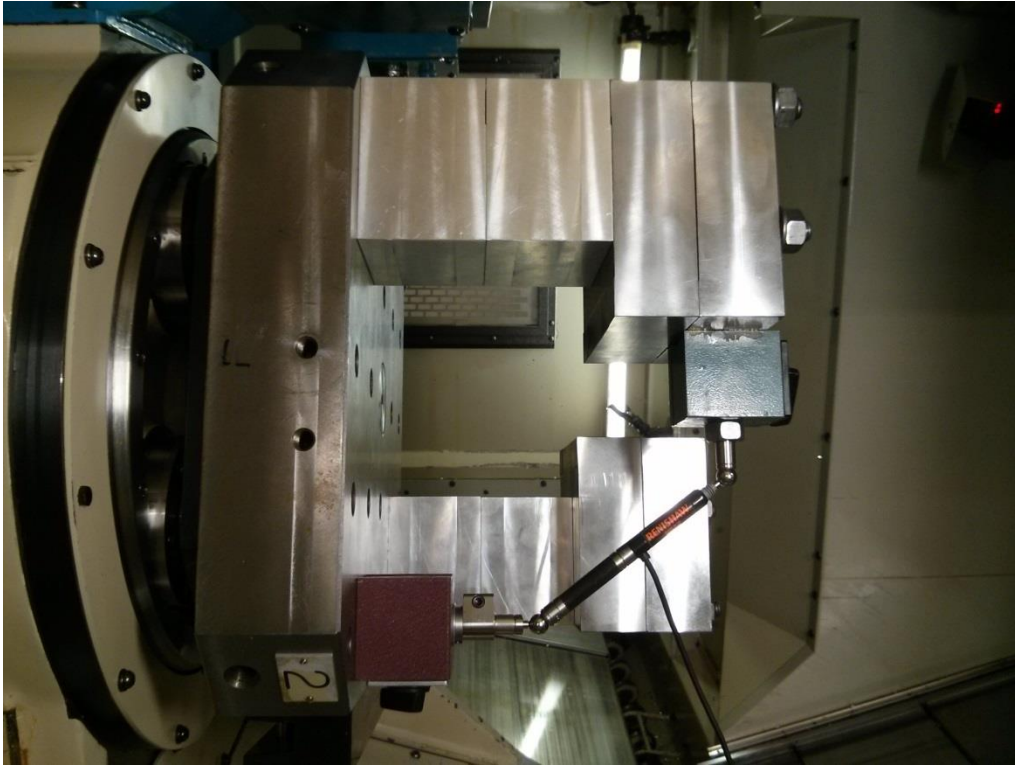


Figure 4-11: The validation test set up with blocks and spacers

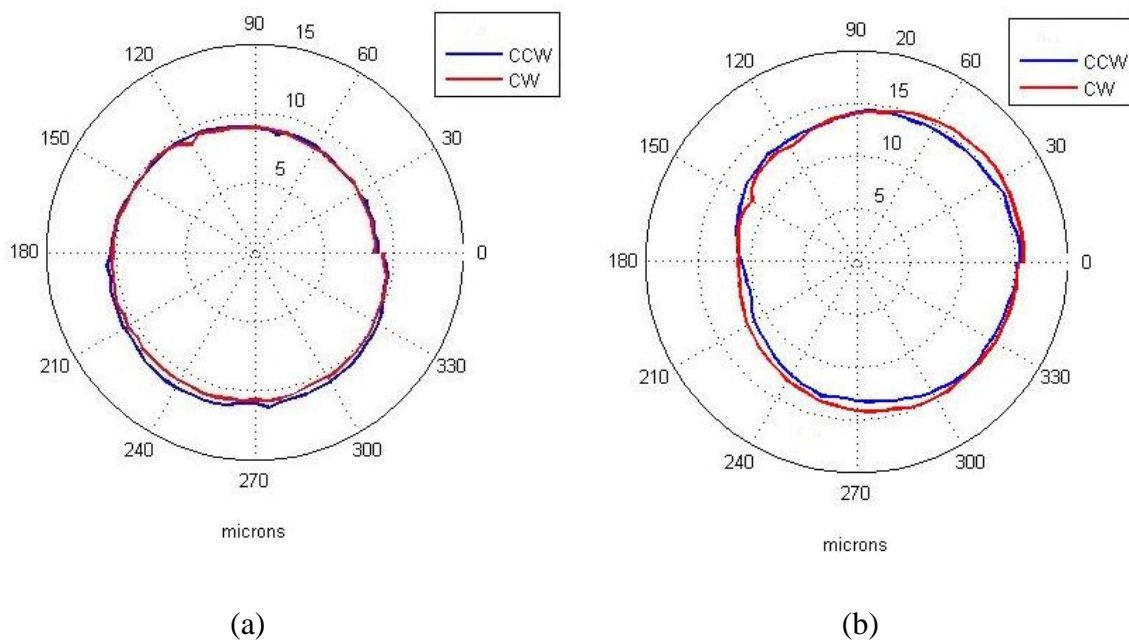


Figure 4-12: The length variation of the magnetic ball bar Renishaw, (a) with only blocks and (b) with blocks and spacers

In figure 4-12 the trends resemble a circle. This indicates that the maximum length variation of the ball bar is 2  $\mu\text{m}$ . Thus, it can be concluded that the deflection of the blocks is insignificant. In the validation test, we did not calculate the deflection of the blocks directly. However, there is a possibility to verify the maximum deflection of the blocks with respect to the length variation of the magnetic ball bar. Figure 4-13 shows the 2D schematic of the validation test. Equation 4-1 presents the relationship between the maximum deflection of the ball bar and the ball bar length variation by assuming that the deflection of the blocks and the deflection of the ball bar are equal.

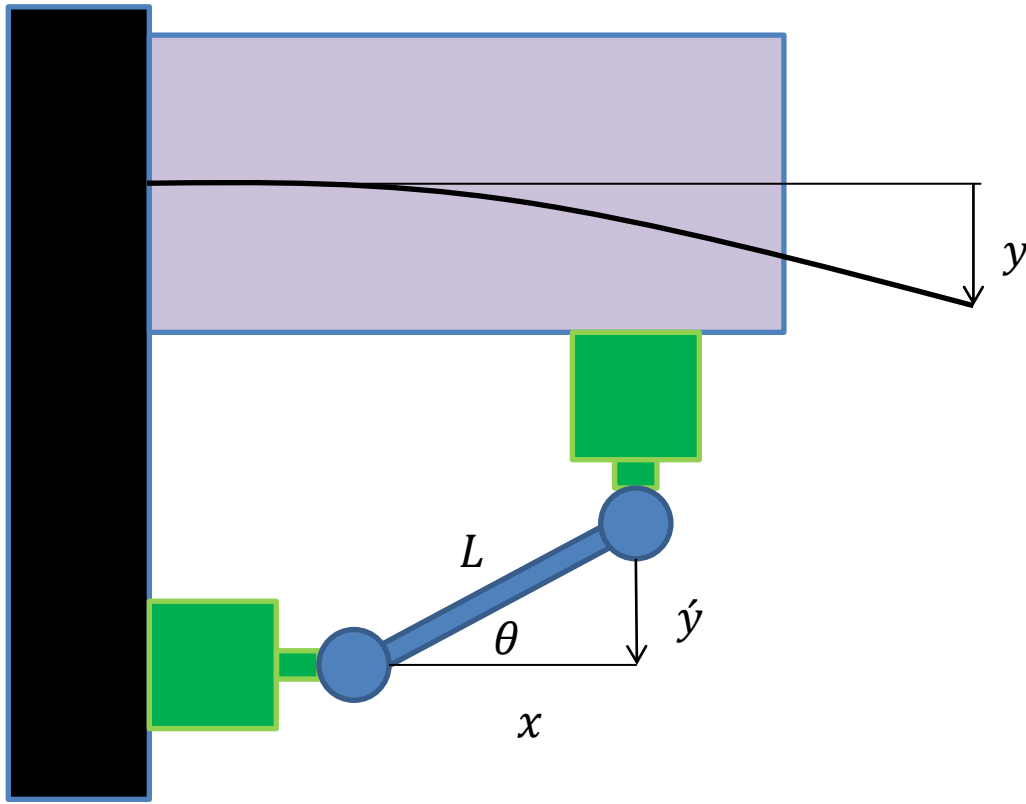


Figure 4-13: The 2D schematic of the validation test

$$\text{if } \hat{y} \cong y \rightarrow L = \sqrt{x^2 + y^2} \rightarrow \frac{dL}{dy} = \frac{y}{\sqrt{x^2 + y^2}} = \sin(\theta) \quad (4-1)$$

As it is shown in equation 4-1, the maximum deflection of the blocks is a function of the angle ( $\theta$ ). Thus, the best set up for the validation tests is that the ball bar is mounted on the machine pallet parallel to the surface of the pallet (vertical). In that case, the ball bar length variation will be equal to the deflection of the ball bar ( $L = y$ ).

## Chapter 5 Analyze the error data

In total, there are 26 motion errors and each error has maximum five coefficients which are called estimated error coefficients. The values of these coefficients are calculated by RUMBA method using the raw data obtained from machine tool. Thus, the polynomial function of each type of the error (motion error) can be achieved by using their coefficients.

### 5.1 Polynomial graphs

The polynomial function can be written as a function of one, two or more arguments. Equation 5-1 indicates the polynomial function 'F' of 'x' argument. In this equation, 'F' is a polynomial function, 'x' is argument, 'n' is a non-negative integer and  $a_0, a_1, a_2, \dots$  are constant coefficients.

$$F(x) = a_0 + a_1x + a_2x^2 + \dots + a_{n-2}x^{n-2} + a_{n-1}x^{n-1} + a_nx^n \quad (5-1)$$

The polynomial functions can have different graphs with respect to their degree. Figure 5-1 indicates the graphs of different degree of polynomial. The graph of a non-constant polynomial always tends to infinity when the variable increases indefinitely [26].

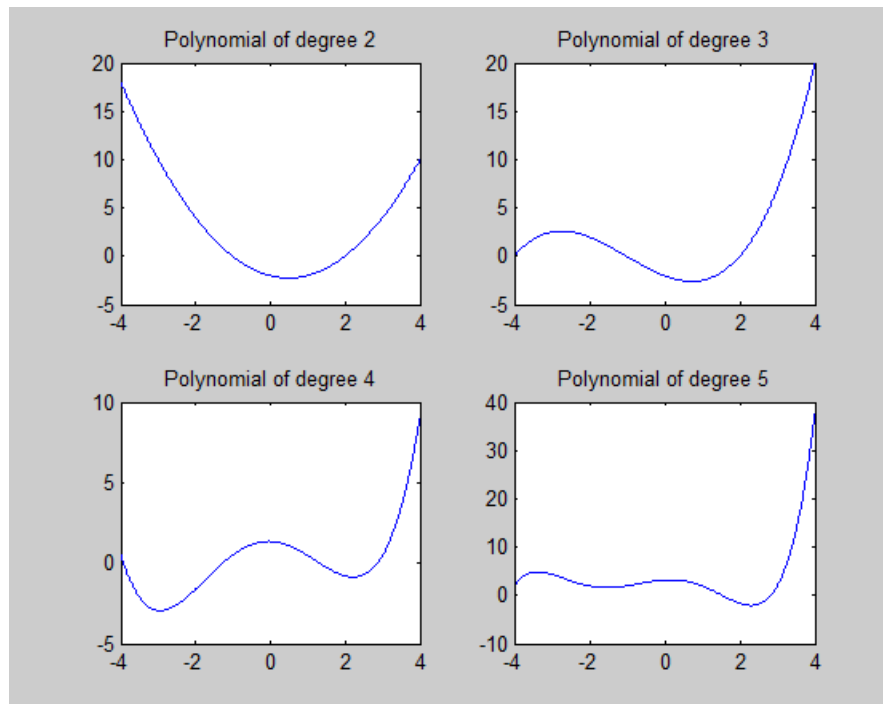


Figure 5-1: Different polynomial graphs

The range of motion errors can be defined as polynomial functions. Each degree of the error indicates one of its error coefficients. For instance, the polynomial function of the error  $EXX$  is defined as equation 5-2:

$$EXX = EXX0 \times X^0 + EXX1 \times X^1 + EXX2 \times X^2 + EXX3 \times X^3 + EXX4 \times X^{+1/-1} \quad (5-2)$$

$EXX0$  is the zero degree error coefficient which is constant and it shows the value of the  $EXX$  regardless to volumetric errors.  $EXX1$  is the first degree error coefficient and it indicates the linear polynomial equation of  $EXX$ .  $EXX2$  is the second degree error coefficient which shows the quadratic polynomial equation of  $EXX$ .  $EXX3$  is the third degree error coefficient and it indicates the cubic polynomial equation of  $EXX$ .  $EXX4$  is the fourth degree error coefficient (which is called backlash error) and it indicates the cubic polynomial equation of  $EXX$ .

All the other range of motion errors follows the same equation (5-2) to define their polynomial function.

Therefore, each range of motion error can be written as a polynomial function by using the values of their coefficients which have been estimated (Appendix II). In other words, each error coefficient of each motion error is estimated by RUMBA using the raw data obtained from the machine and then entered in polynomial equation. The polynomial graphs of each motion error in different set up (experimental tests) are sketched and overlaid to show how the range of the error changes for different loading conditions. Thus, it is possible to follow the trends of each range of error with respect to different weights and torques. Plotting polynomial curves have been programmed in Matlab (Appendix I).

The following figures show the polynomial graphs of each range of motion error for different weights and torques. To have a better understanding of how the error changes, the rainbow colors have been used. They start with color blue and end with color red. The order of the colors is presented below and also shows graphically in Figure 5-2:

1. Blue continues : test1 (W=0N, T=0N/m)
2. Blue dash : test2 (W=200N, T=5N/m)
3. Cyan continues : test10 (W=200N, T=20N/m)
4. Cyan dash : test12 (W=200N, T=35.2N/m)

5. Green continues : test7 (W=400N, T=10N/m)
6. Green dash : test3 (W=400N, T=20N/m)
7. Yellow continues : test4 (W=600N, T=45N/m)
8. Yellow dash : test11 (W=600N, T=90N/m)
9. Magenta continues : test8 (W=800N, T=40N/m)
10. Magenta dash : test5 (W=800N, T=80N/m)
11. Red continues : test9 (W=1000N, T=75N/m)
12. Red dash : test6 (W=1000N, T=125N/m)

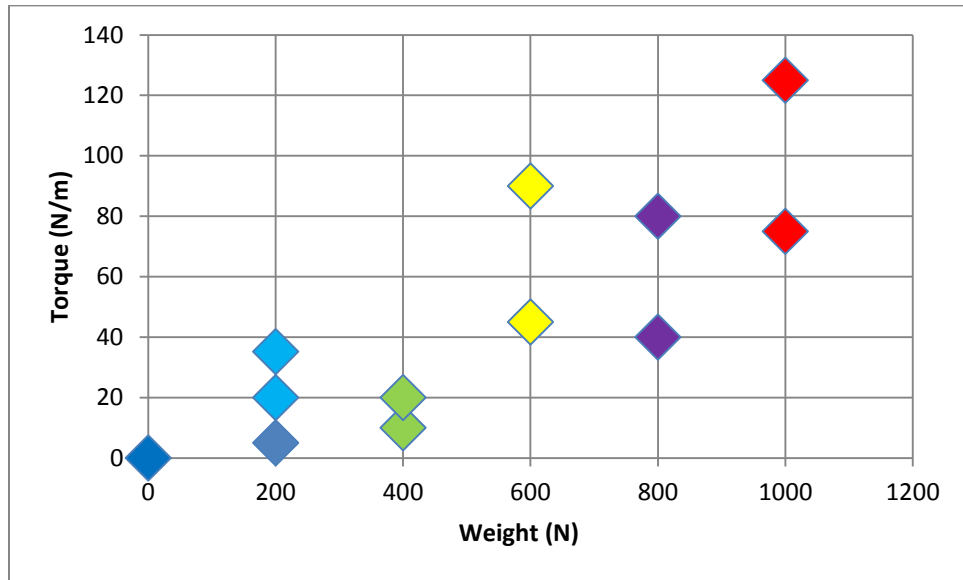


Figure 5-2: The rainbow colors of the polynomial graphs

In these figures, the horizontal axis shows the range of the corresponding joint positioning. These ranges can be defined from the probing file. To this aim, the maximum and the minimum values of each axis should be considered.

- The range of the X axis is equal to [-300 mm, 300 mm].
- The range of the Y axis is equal to [-250 mm, 250 mm].
- The range of the Z axis is equal to [0 mm, 305 mm].
- The range of the A, B and C axes are equal to  $[0^\circ, 360^\circ]$ ,  $[-90^\circ, 90^\circ]$  and  $[-180^\circ, 180^\circ]$  respectively.

$$EXX = EXX0 \times X^0 + EXX1 \times X^1 + EXX2 \times X^2 + EXX3 \times X^3 + EXX4 \times X^{+1/-1}$$

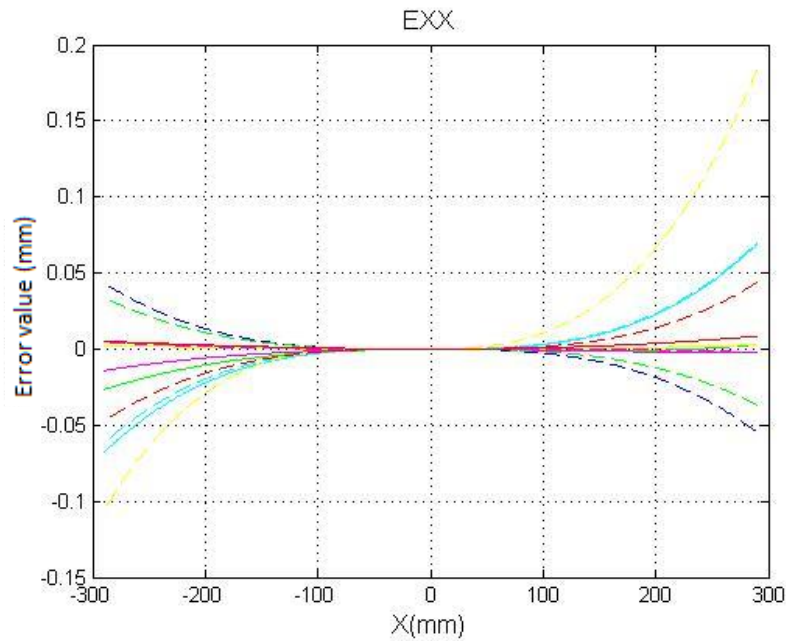


Figure 5-3: The polynomial graphs of the error EXX

At X equal to zero there is no EXX error. In other words, when the machine pallet doesn't move in X direction, the EXX error is zero; but as soon as the pallet moves in X, the EXX error will be created.

The graphs are quite similar which have ascending trajectory in large weights and torques and descending trajectory in small values of weights and torques. If we follow the colors from color blue to color red, the value of the error generally decreases by adding more load on the machine.

The absolute value of the error in each graph is symmetrical and that is expected. The maximum error value is around 0.2 mm and the minimum absolute value is 0 mm.

$$EYX = EYX0 \times X^0 + EYX1 \times X^1 + EYX2 \times X^2 + EYX3 \times X^3 + EYX4 \times X^{+1/-1}$$

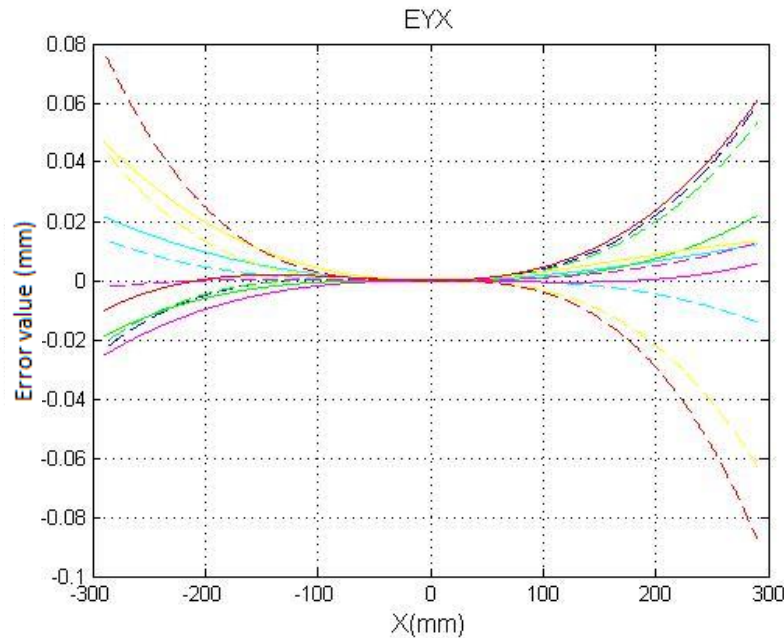


Figure 5-4: The polynomial graphs of the error EYX

There is no EYX error at  $X=0\text{mm}$ . It means that the value of the error is equal to zero or the effect of load neutralized this type of motion error at  $X$  equal to zero.

Some of the graphs have ascending trajectory and some other have descending trajectory in different loading conditions.

From color blue to color red, the value of the error generally increases by increasing weight and torque.

The maximum value is close to 0.08 mm and the minimum absolute value is 0.

$$EZX = EZX0 \times X^0 + EZX1 \times X^1 + EZX2 \times X^2 + EZX3 \times X^3 + EZX4 \times X^{+1/-1}$$

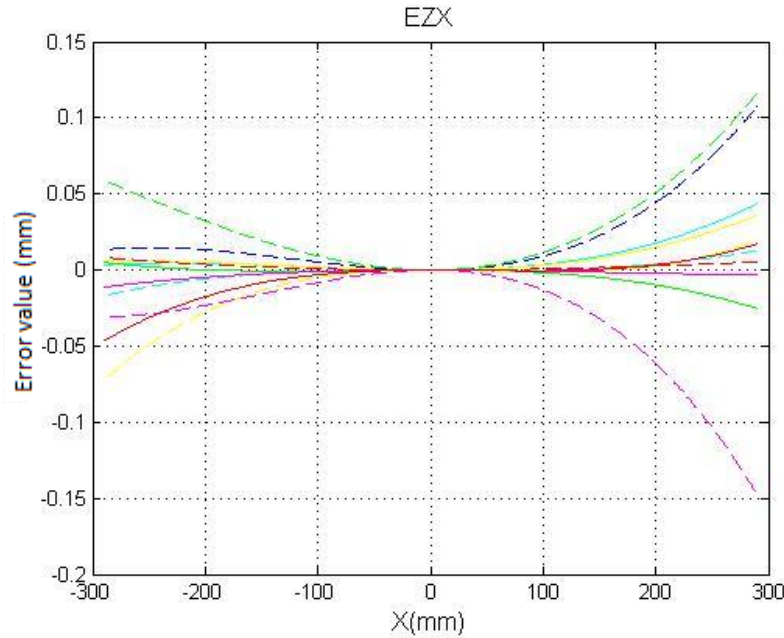


Figure 5-5: The polynomial graphs of the error EZX

There is no EZX error at  $X=0\text{mm}$ . It means that the value of the error is equal to zero or the effect of load neutralized this type of motion error at  $X$  equal to zero.

Most of the graphs have an ascending trajectory in the range of  $X$  axis. In other words, when the machine pallet moves from  $X (-)$  to  $X (+)$ , the value of the EZX motion error will increase.

From color blue to color red, the error value generally decreases by adding more load on the machine.

The maximum error value is 0.15 mm and the minimum absolute value is 0.



$$EAX = EAX0 \times X^0 + EAX1 \times X^1 + EAX2 \times X^2 + EAX3 \times X^3 + EAX4 \times X^{+1/-1}$$

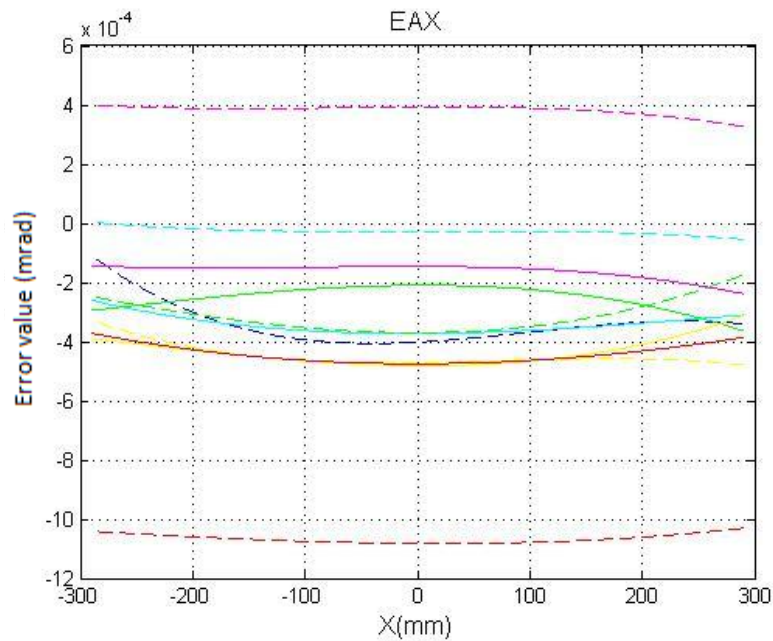


Figure 5-6: The polynomial graphs of the error EAX

If we follow the colors from color blue to color red (which belong to minimum weight and torque and maximum weight and torque respectively), the value of the error generally increases by adding more load on the machine.

The trends of the graphs are quite constant in the range of the X axis position. It means that the value of the EAX error doesn't change significantly from X (–) to X (+).

The maximum absolute value of the error is close to 0.001 mm and the minimum absolute value is 0.

$$EBX = EBX0 \times X^0 + EBX1 \times X^1 + EBX2 \times X^2 + EBX3 \times X^3 + EBX4 \times X^{+1/-1}$$

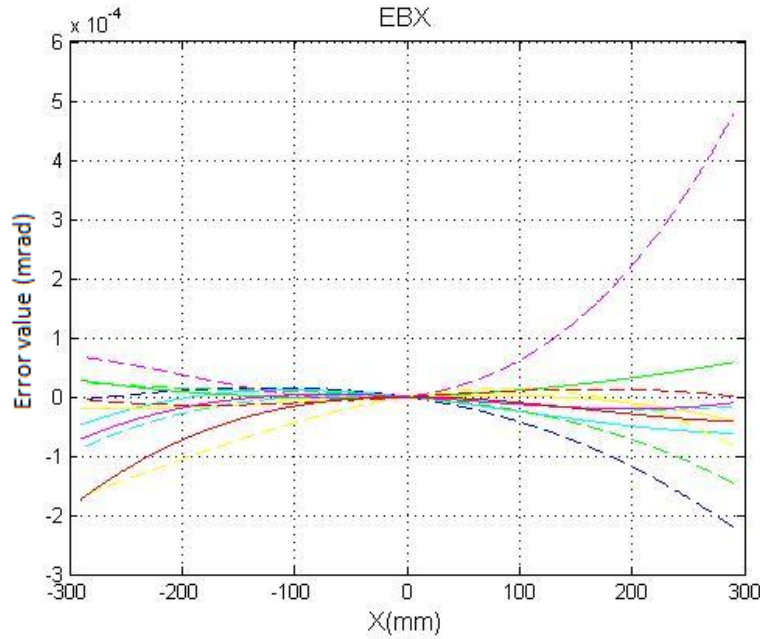


Figure 5-7: The polynomial graphs of the error EBX

There is no EYX error at  $X=0\text{mm}$ . It means that the value of the error is equal to zero or the effect of load neutralized this type of motion error at  $X$  equal to zero.

The value of the error in the range of positive  $X$  is larger than the error value in the range of negative  $X$ .

The value of the EBX motion error doesn't change significantly by adding more load on the machine. Thus, it can be concluded that adding load on the machine doesn't affect this type of motion error significantly.

The maximum error value is  $0.0005\text{ mm}$  or  $0.5\text{ }\mu\text{m}$ .

$$ECX = ECX0 \times X^0 + ECX1 \times X^1 + ECX2 \times X^2 + ECX3 \times X^3 + ECX4 \times X^{+1/-1}$$

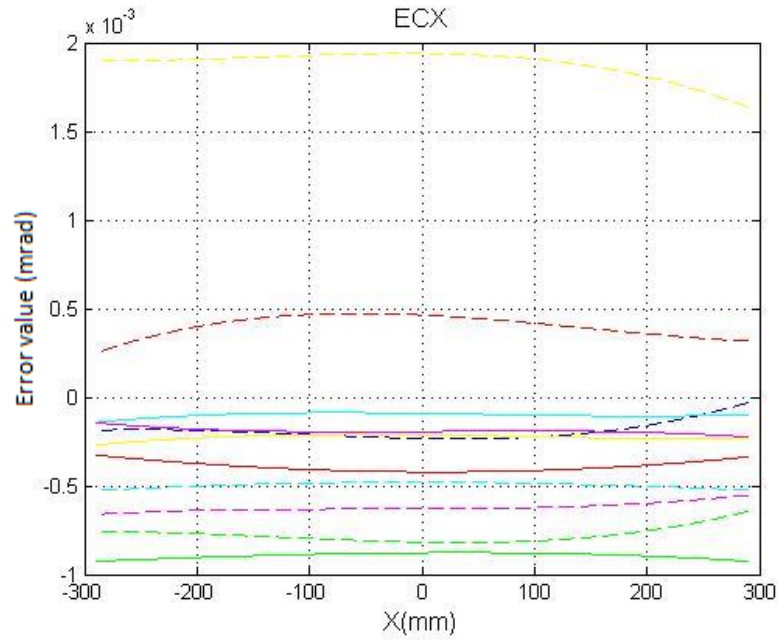


Figure 5-8: The polynomial graphs of the error ECX

The trends of the graphs are quite constant in the range of the X axis position. It means that the value of the ECX error doesn't change significantly from X (–) to X (+).

From color blue to color red, the error value generally increases by adding more load on the machine.

The maximum value of the error is equal to 0.002 mm and the minimum absolute value is 0 mm.

$$EXY = EXY0 \times Y^0 + EXY1 \times Y^1 + EXY2 \times Y^2 + EXY3 \times Y^3 + EXY4 \times Y^{+1/-1}$$

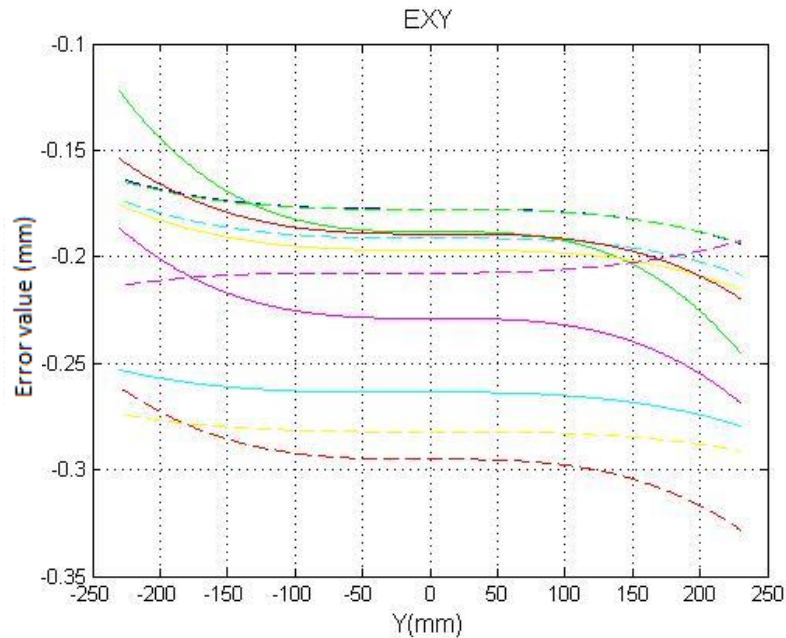


Figure 5-9: The polynomial graphs of the error EXY

If we follow the colors from color blue to color red (which belong to minimum weight and torque and maximum weight and torque respectively), the value of the error generally increases by adding more load on the machine.

All the graphs follow the same trend in the range of Y axis. They have a completely ascending trajectory according to the absolute values.

The maximum absolute error value is around 0.33 mm and the minimum absolute value is close to 0.12 mm.

$$EYY = EYY0 \times Y^0 + EYY1 \times Y^1 + EYY2 \times Y^2 + EYY3 \times Y^3 + EYY4 \times Y^{+1/-1}$$

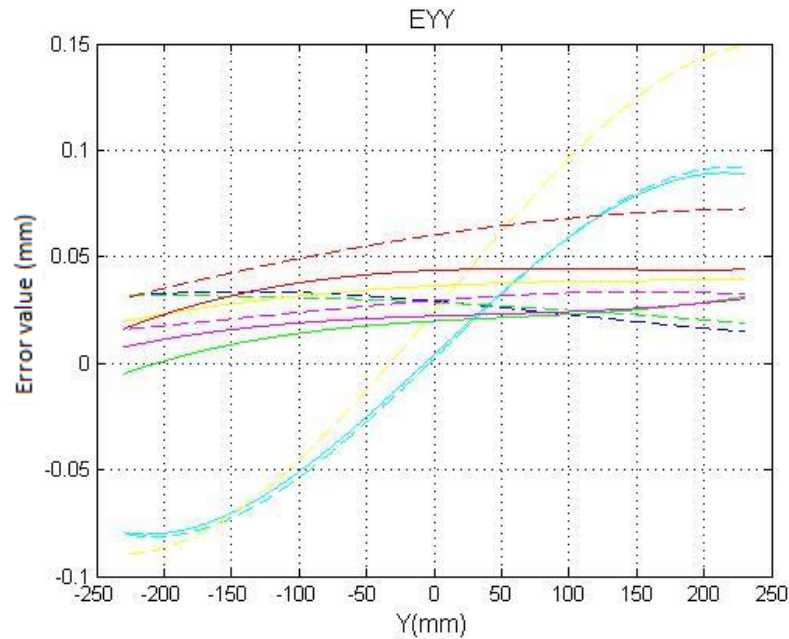


Figure 5-10: The polynomial graphs of the error EYY

The value of the error increases from negative Y to positive Y.

From color blue to color red, the error value generally increases by adding more load on the machine.

Most of the graphs have similar trend (quite constant in the range of Y axis). However, some graphs such as cyan continues, cyan dash and yellow dash have different patterns.

The maximum error value is 0.15mm and the minimum absolute value is 0 mm.

$$EZY = EZY0 \times Y^0 + EZY1 \times Y^1 + EZY2 \times Y^2 + EZY3 \times Y^3 + EZY4 \times Y^{+1/-1}$$

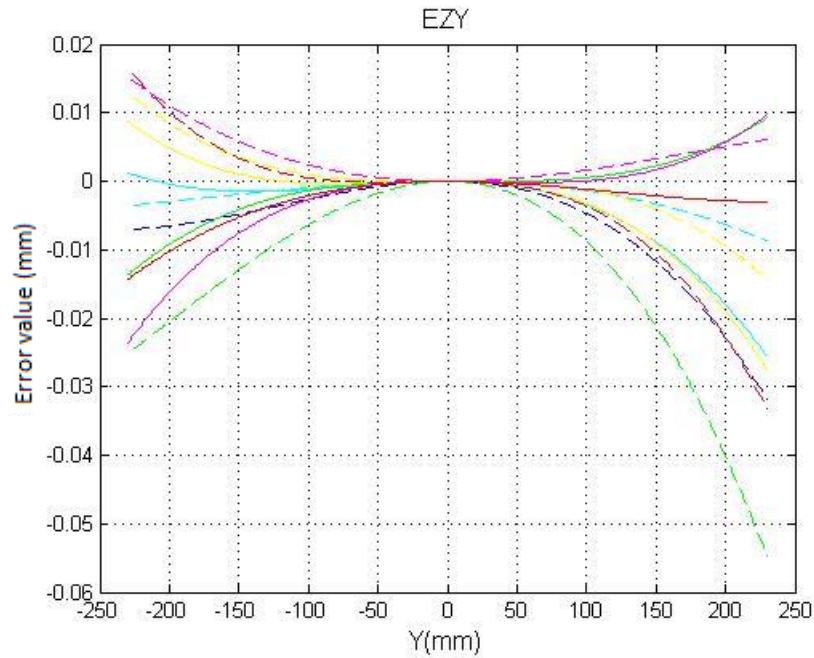


Figure 5-11: The polynomial graphs of the error EZY

There is no EZY error at  $Y=0\text{mm}$ . It means that the value of the error is equal to zero or the effect of load neutralized this type of motion error at  $Y$  equal to zero.

Most of the graphs have the same trend which is quadratic. It means that the EZY motion error is symmetric in the range of  $Y$  axis.

From color blue to color red, the error value generally decreases by adding more load on the machine.

The maximum absolute error value is close to 0.055 mm.

$$EXZ = EXZ0 \times Z^0 + EXZ1 \times Z^1 + EXZ2 \times Z^2 + EXZ3 \times Z^3 + EXZ4 \times Z^{+1/-1}$$

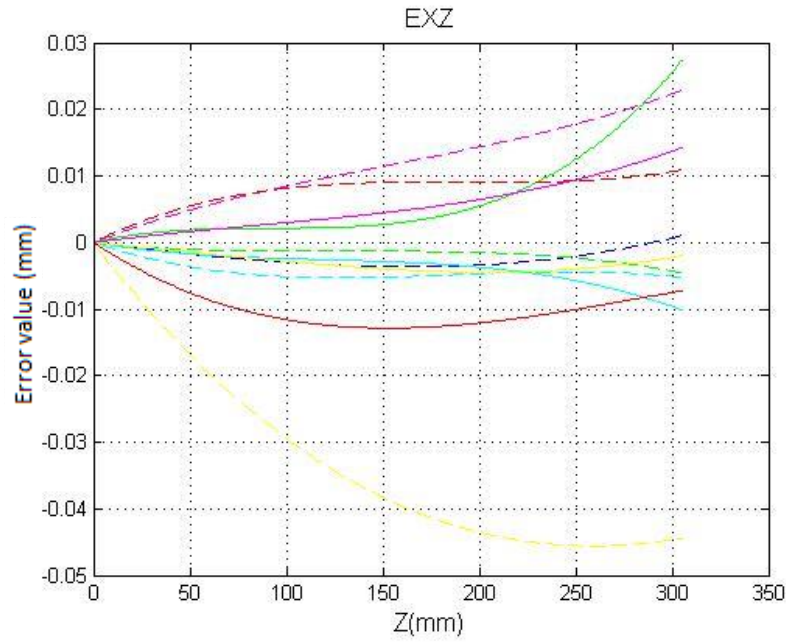


Figure 5-12: The polynomial graphs of the error EXZ

There is no EXZ error at  $Z=0$ mm. It means that the value of the error is equal to zero or the effect of load neutralized this type of motion error at  $Z$  equal to zero.

All the graphs have ascending trajectory in the range of  $Z$  axis position. In other words, the error values increase by increasing the value of  $Z$  axis.

From color blue to color red, the error value generally increases by adding more load on the machine.

The maximum error value is close to 0.03 mm.

$$EYZ = EYZ0 \times Z^0 + EYZ1 \times Z^1 + EYZ2 \times Z^2 + EYZ3 \times Z^3 + EYZ4 \times Z^{+1/-1}$$

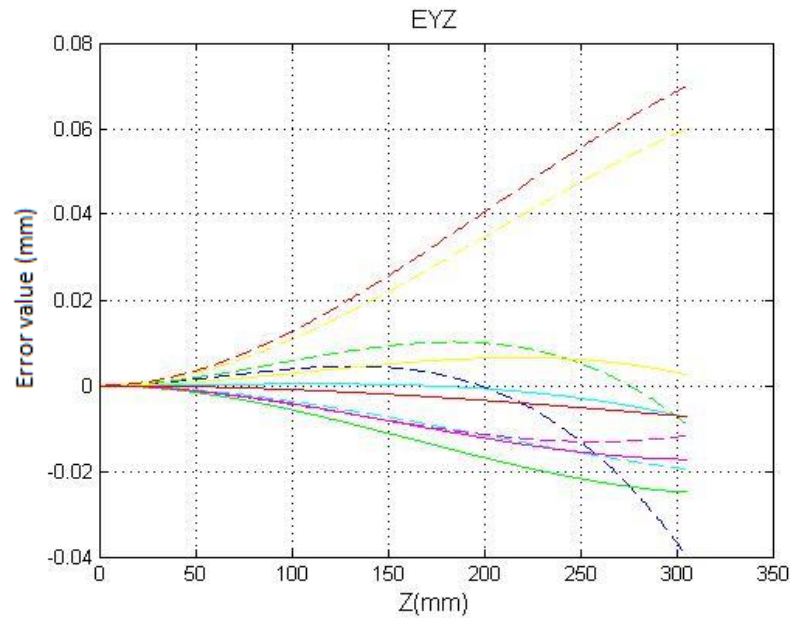


Figure 5-13: The polynomial graphs of the error EYZ

Most of the graphs have the same trend. However, some graphs (magenta dash and yellow dash) have different trajectory.

There is no EYZ error at  $Z=0\text{mm}$ . It means that the value of the error is equal to zero or the effect of load neutralized this type of motion error at  $Z$  equal to zero.

Adding more loads on the machine pallet doesn't have any significant effect on the error values.

The maximum error value is close to 0.07 mm.



$$EZZ = EZZ0 \times Z^0 + EZZ1 \times Z^1 + EZZ2 \times Z^2 + EZZ3 \times Z^3 + EZZ4 \times Z^{+1/-1}$$

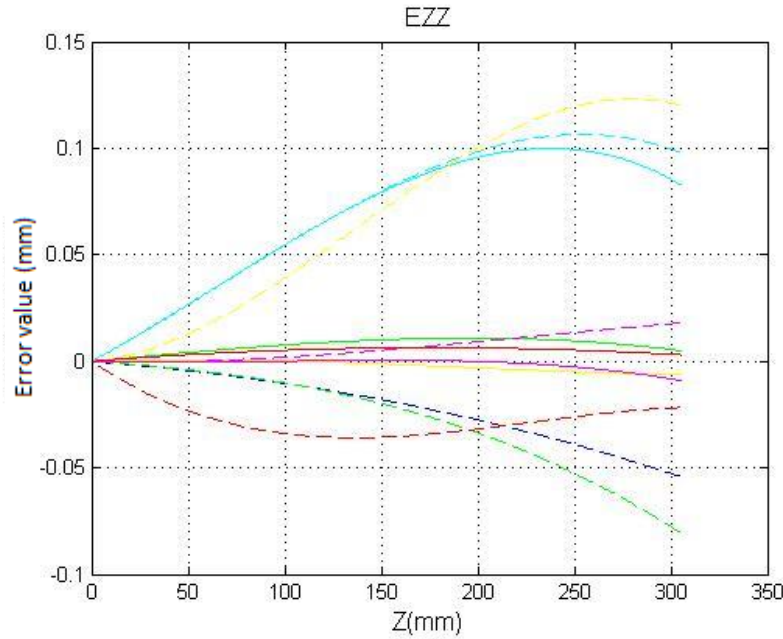


Figure 5-14: The polynomial graphs of the error EZZ

There is no EZZ error at  $Z=0\text{mm}$ . It means that the value of the error is equal to zero or the effect of load neutralized this type of motion error at  $Z$  equal to zero.

The range of the error value in large weights and torques are quite constant in  $Z$  axis position.

If we follow the colors from color blue to color red (which belong to minimum weight and torque and maximum weight and torque respectively), the value of the error generally decreases by adding more load on the machine.

$$E_{AZ} = E_{AZ0} \times Z^0 + E_{AZ1} \times Z^1 + E_{AZ2} \times Z^2 + E_{AZ3} \times Z^3 + E_{AZ4} \times Z^{+1/-1}$$

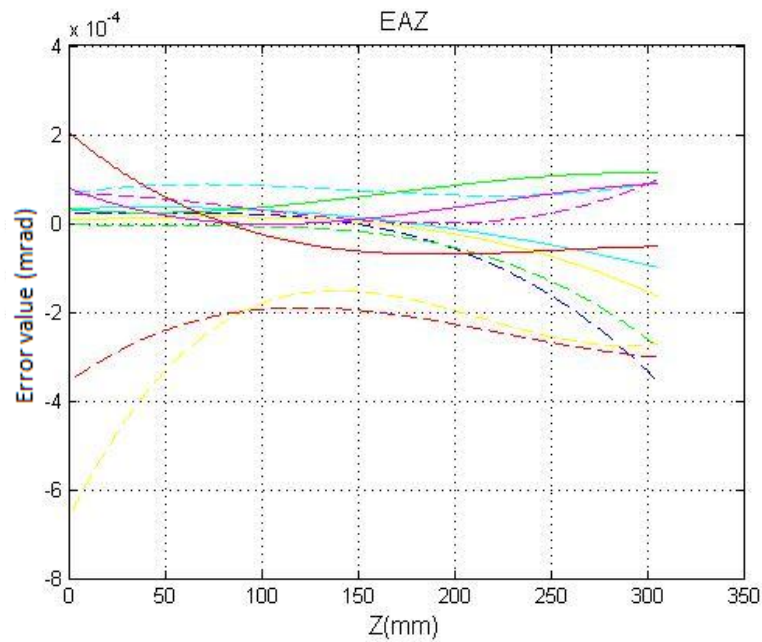


Figure 5-15: The polynomial graphs of the error EAZ

The values of the error in small weights and torques at Z equal to zero are insignificant, but in large weights, they start from -0.0006 mm.

The error values generally decrease in the range of Z axis. In other words, when the probe moves forward to approach the master balls artifact, the EAZ value will decrease.

The maximum absolute error value is around 0.0006 mm.

$$ECZ = ECZ0 \times Z^0 + ECZ1 \times Z^1 + ECZ2 \times Z^2 + ECZ3 \times Z^3 + ECZ4 \times Z^{+1/-1}$$

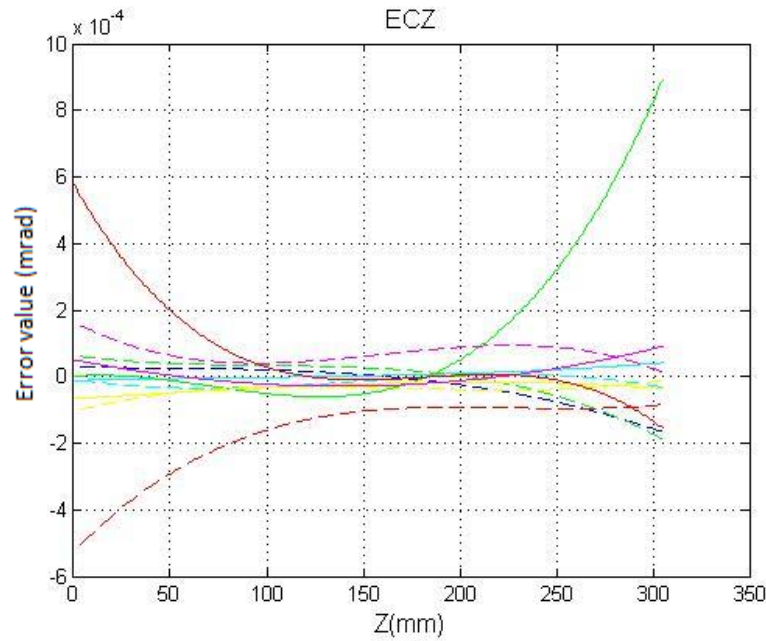


Figure 5-16: The polynomial graphs of the error ECZ

Most of the graphs (except magenta dash and green continues) have the same trend and their value is close to zero in the range of Z axis position.

Adding more loads on the machine pallet doesn't have any specific effect on the error value in most of the loading conditions.

The maximum absolute value of the error is around 0.0009 mm or 0.9  $\mu\text{m}$ .

$$EXB = EXB0 \times B^0 + EXB1 \times B^1 + EXB2 \times B^2 + EXB3 \times B^3 + EXB4 \times B^{+1/-1}$$

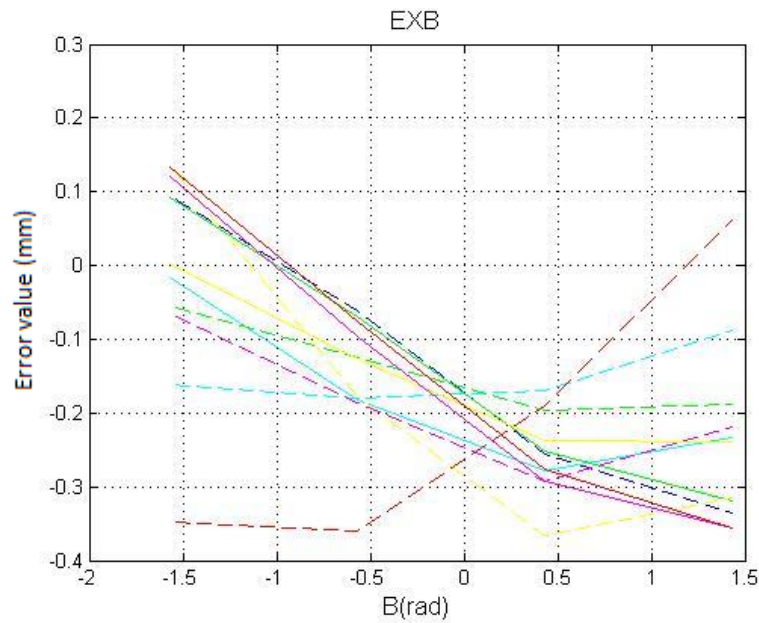


Figure 5-17: The polynomial graphs of the error EXB

All the graphs have an ascending trajectory (except magenta dash) in the range of B axis. In other words, the absolute error values generally decreases from  $B = -90^\circ$  to  $B = 90^\circ$ .

The graphs are quite close to each other in the range of B axis. It means that adding more loads on the machine doesn't have significant effect on the EXB error value.

The maximum absolute error value is around 0.35 mm.

$$EYB = EYB0 \times B^0 + EYB1 \times B^1 + EYB2 \times B^2 + EYB3 \times B^3 + EYB4 \times B^{+1/-1}$$

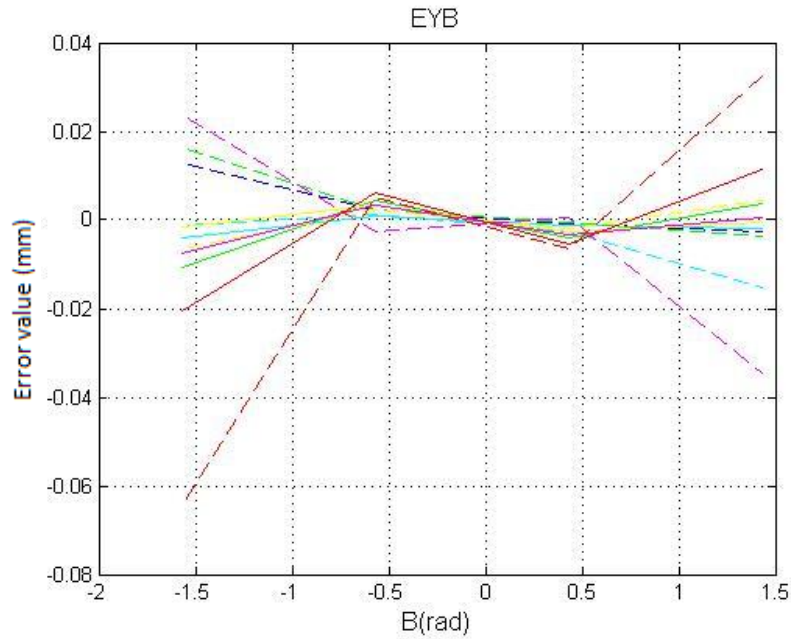


Figure 5-18: The polynomial graphs of the error EYB

All the graphs have the same trend. From color blue to color red, the error value generally increases by adding more load on the machine.

The absolute values of the error decrease in B negative and increase in B positive in most loading conditions.

Magenta dash which belongs to test number 5 ( $W=800N$ ,  $T=80N/m$ ) changes much more than the other graphs in the range of B axis.

The maximum error value is around 0.06 mm.

$$EZB = EZB0 \times B^0 + EZB1 \times B^1 + EZB2 \times B^2 + EZB3 \times B^3 + EZB4 \times B^{+1/-1}$$

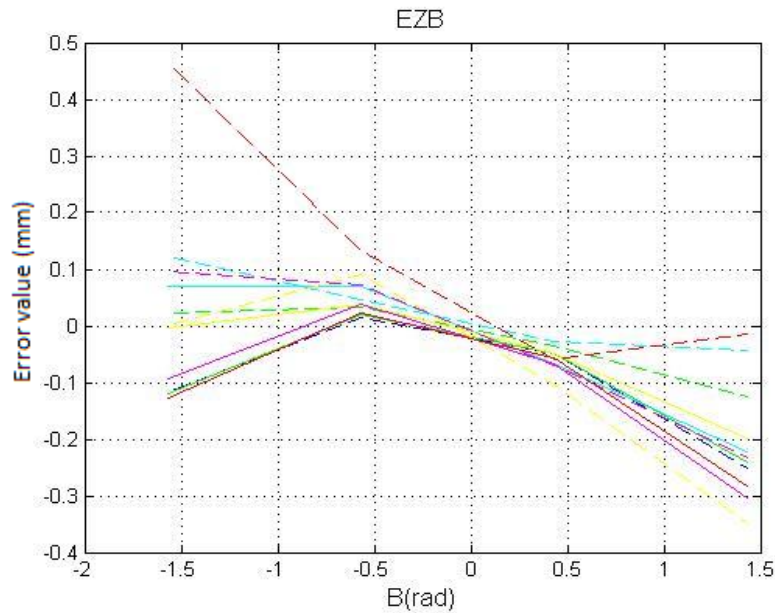


Figure 5-19: The polynomial graphs of the error EZB

Most of the graphs have the same trend except red dash which belongs to maximum weight and maximum torque.

The graphs are quite close to each other in the range of B axis. It means that adding more loads on the machine doesn't have significant effect on the EXB error value.

The maximum error value is around 0.45 mm.

$$EAB = EAB0 \times B^0 + EAB1 \times B^1 + EAB2 \times B^2 + EAB3 \times B^3 + EAB4 \times B^{+1/-1}$$

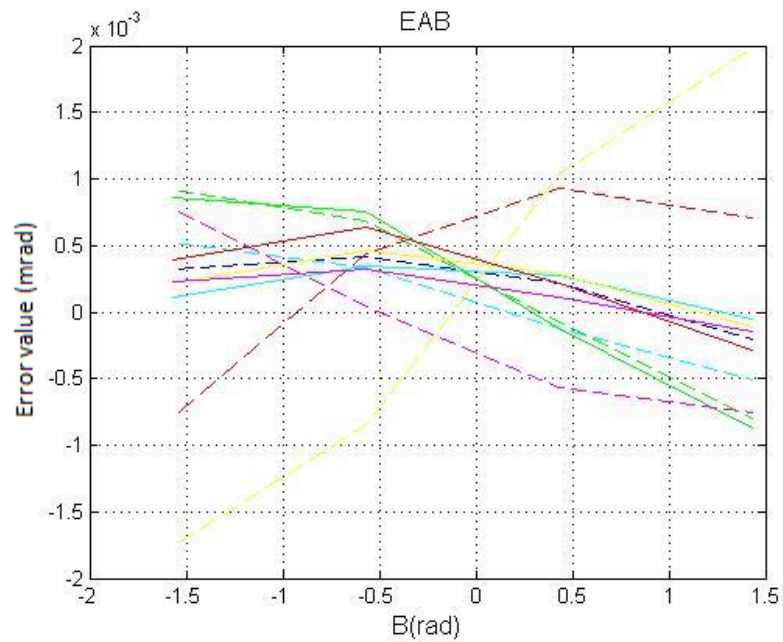


Figure 5-20: The polynomial graphs of the error EAB

Most of the graphs have the same trend. They have a descending trajectory in the range of B axis of the machine tool.

If we follow the colors from color blue to color red (which belong to minimum weight and torque and maximum weight and torque respectively), the value of the error generally increases by adding more load on the machine.

The maximum error value is around 0.002 mm.

$$EBB = EBB0 \times B^0 + EBB1 \times B^1 + EBB2 \times B^2 + EBB3 \times B^3 + EBB4 \times B^{+1/-1}$$

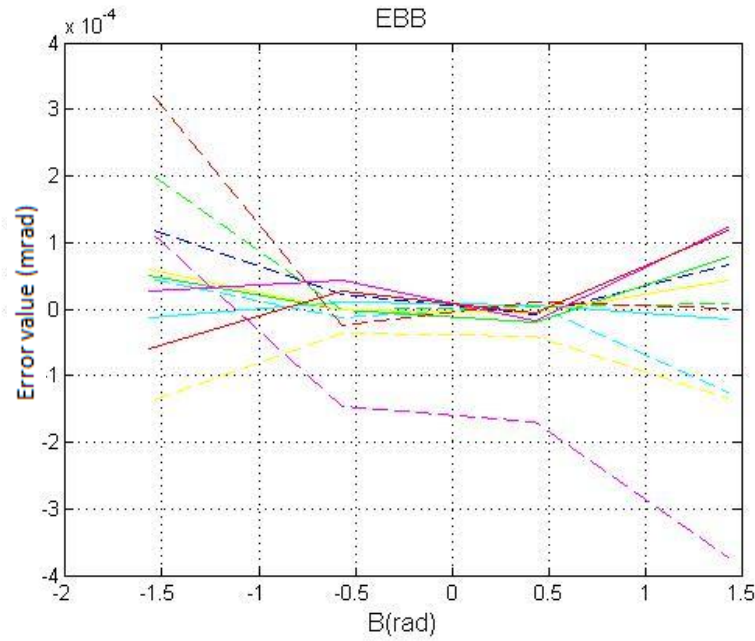


Figure 5-21: The polynomial graphs of the error EBB

From color blue to color red, the error value generally increases by adding more load on the machine.

The graphs have the same trend (descending in B negative, constant close to B zero and then ascending in B positive).

Most of the error values in different loading conditions are close to zero at B equal to zero.

The maximum absolute error value is around 0.0004 mm.



$$ECB = ECB0 \times B^0 + ECB1 \times B^1 + ECB2 \times B^2 + ECB3 \times B^3 + ECB4 \times B^{+1/-1}$$

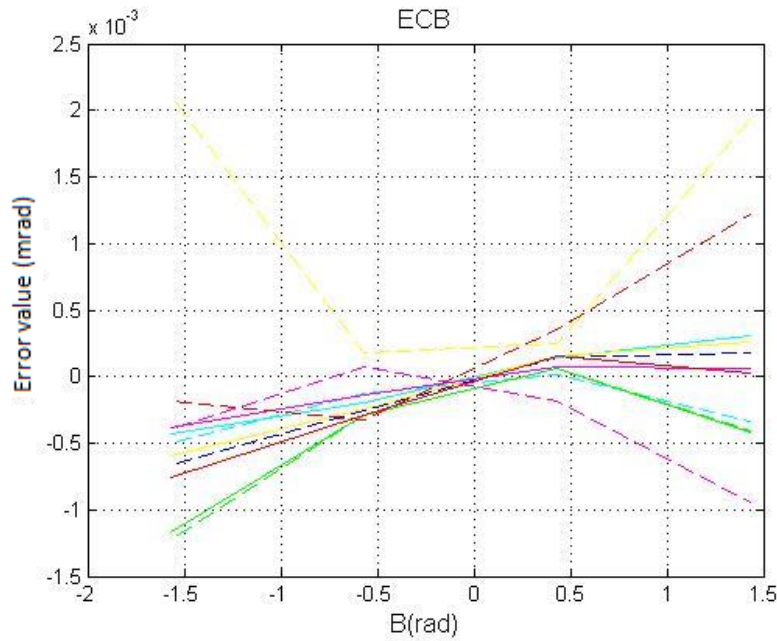


Figure 5-22: The polynomial graphs of the error ECB

From color blue to color red, the error value generally increases by adding more load on the machine.

Most of the error values in different loading conditions are close to zero at B equal to zero.

The maximum error value is around 0.0012 mm or 1.2  $\mu\text{m}$ .

$$EXC = EXC0 \times C^0 + EXC1 \times C^1 + EXC2 \times C^2 + EXC3 \times C^3 + EXC4 \times C^{+1/-1}$$

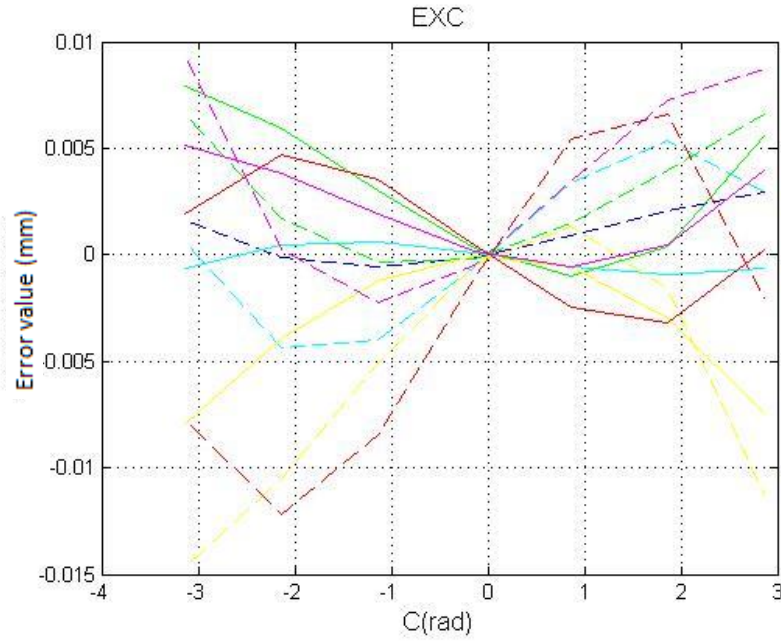


Figure 5-23: The polynomial graphs of the error EXC

There is no EXC error at C equal to zero. In other words, when the machine pallet doesn't rotate around its C axis, the EXC error is zero; but as soon as the pallet rotates around C, the EXC error will be created.

From color blue to color red, the error value generally increases by adding more load on the machine.

Each graph has a minimum value and a maximum value in C (−) and C (+) respectively.

The maximum absolute error value is around 0.012 mm.

$$EYC = EYC0 \times C^0 + EYC1 \times C^1 + EYC2 \times C^2 + EYC3 \times C^3 + EYC4 \times C^{+1/-1}$$

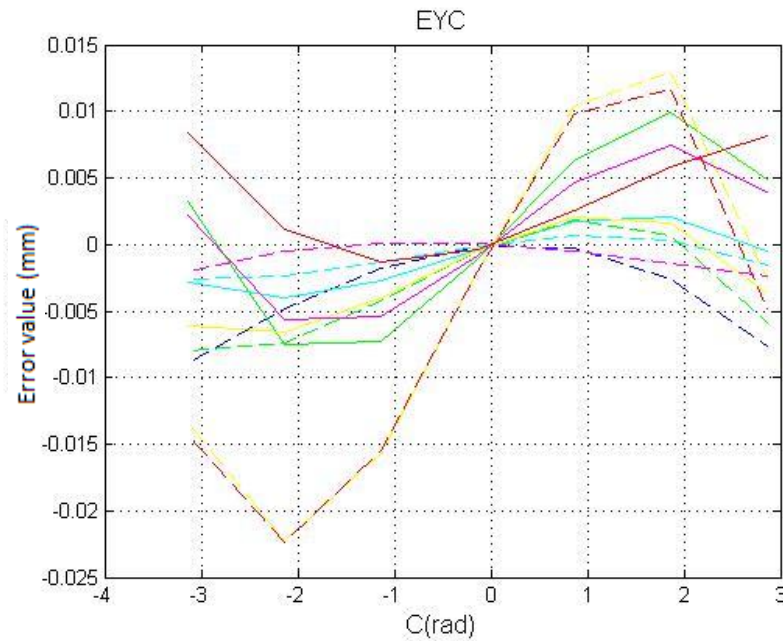


Figure 5-24: The polynomial graphs of the error EYC

There is no EYC error at C equal to zero. In other words, when the machine pallet doesn't rotate around its C axis, the EYC error is zero; but as soon as the pallet rotates around C, the EYC error will be created.

From color blue to color red, the error value generally increases by adding more load on the machine.

Most of the graphs follow the same trend except yellow continues and red dash.

The maximum absolute error value is around 0.022mm.

$$EYC = EYC0 \times C^0 + EYC1 \times C^1 + EYC2 \times C^2 + EYC3 \times C^3 + EYC4 \times C^{+1/-1}$$

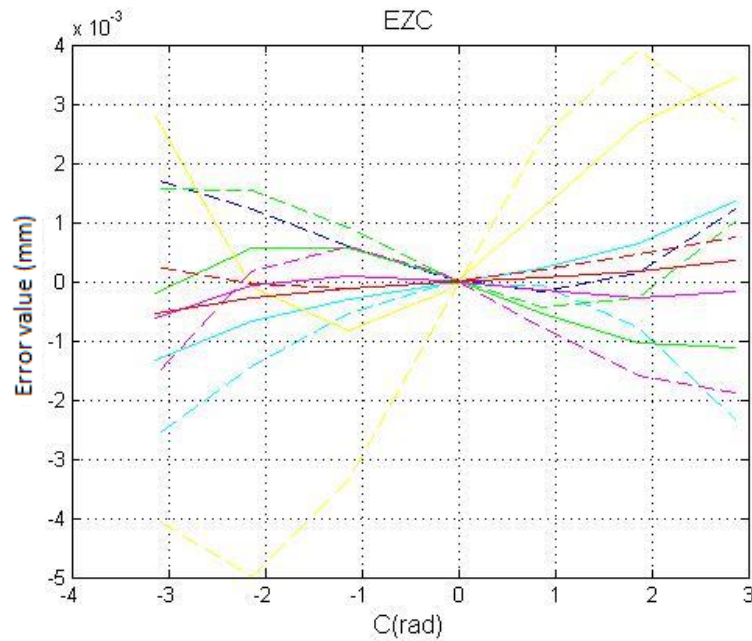


Figure 5-25: The polynomial graphs of the error EZC

There is no EZC error at C equal to zero. In other words, when the machine pallet doesn't rotate around its C axis, the EZC error is zero; but as soon as the pallet rotates around C, the EZC error will be created.

From color blue to color red, the error value generally decreases by adding more load on the machine.

All the graphs except yellow continue and yellow dash, have the same trend.

The maximum error value is around 0.005 mm.

$$EAC = EAC0 \times C^0 + EAC1 \times C^1 + EAC2 \times C^2 + EAC3 \times C^3 + EAC4 \times C^{+1/-1}$$

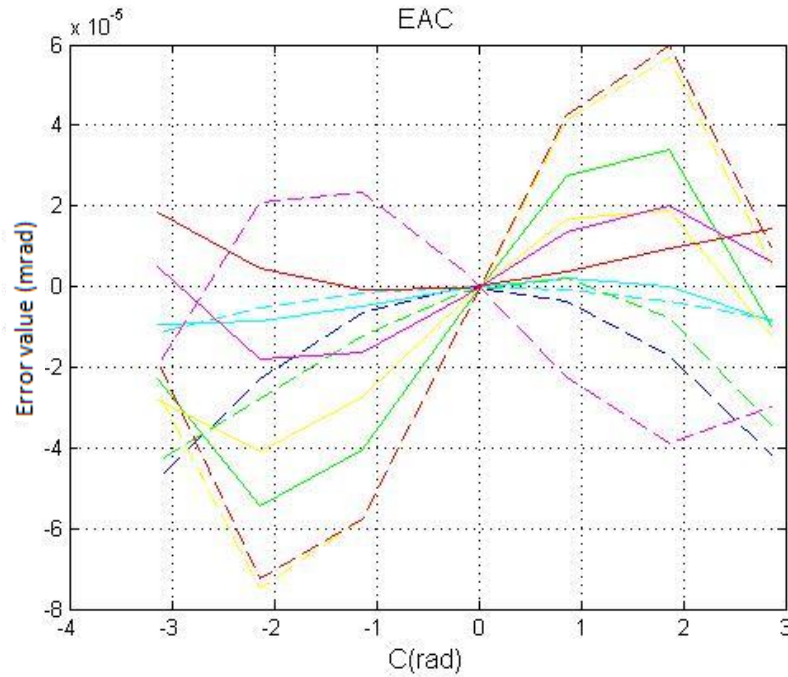


Figure 5-26: The polynomial graphs of the error EAC

There is no EAC error at C equal to zero. In other words, when the machine pallet doesn't rotate around its C axis, the EAC error is zero; but as soon as the pallet rotates around C, the EAC error will be created.

From color blue to color red, the error value generally increases by adding more load on the machine.

The error values of EAC are much smaller than the other types of errors. Thus, this motion error is negligible.

The maximum error value is around 0.00006 mm or 0.06  $\mu\text{m}$ .

$$EBC = EBC0 \times C^0 + EBC1 \times C^1 + EBC2 \times C^2 + EBC3 \times C^3 + EBC4 \times C^{+1/-1}$$

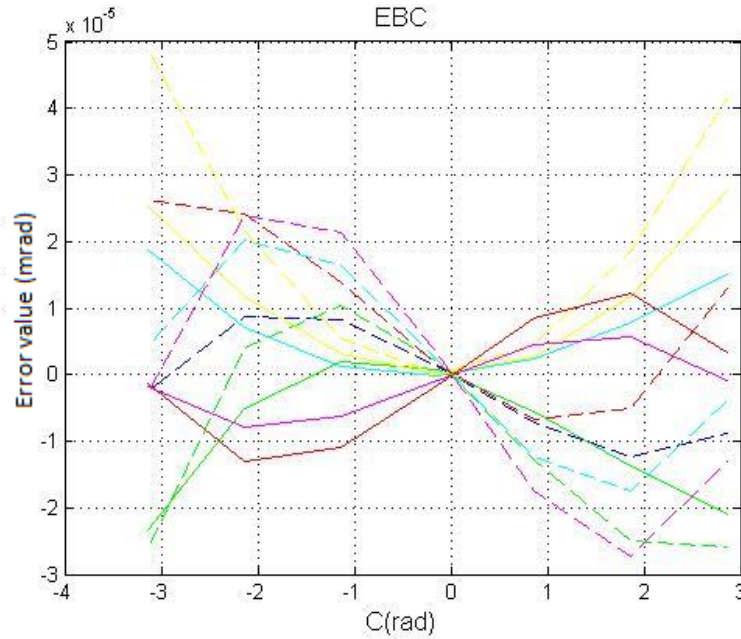


Figure 5-27: The polynomial graphs of the error EBC

There is no EBC error at C equal to zero. In other words, when the machine pallet doesn't rotate around its C axis, the EBC error is zero; but as soon as the pallet rotates around C, the EBC error will be created.

From color blue to color red, the error value generally increases by adding more load on the machine.

All the graphs except yellow continue and yellow dash has the same trend.

The error values of EBC error are much smaller than the other types of the errors. Thus, it can be negligible.

The maximum error value is around 0.00005 mm or 0.05  $\mu\text{m}$ .

$$ECC = ECC0 \times C^0 + ECC1 \times C^1 + ECC2 \times C^2 + ECC3 \times C^3 + ECC4 \times C^{+1/-1}$$

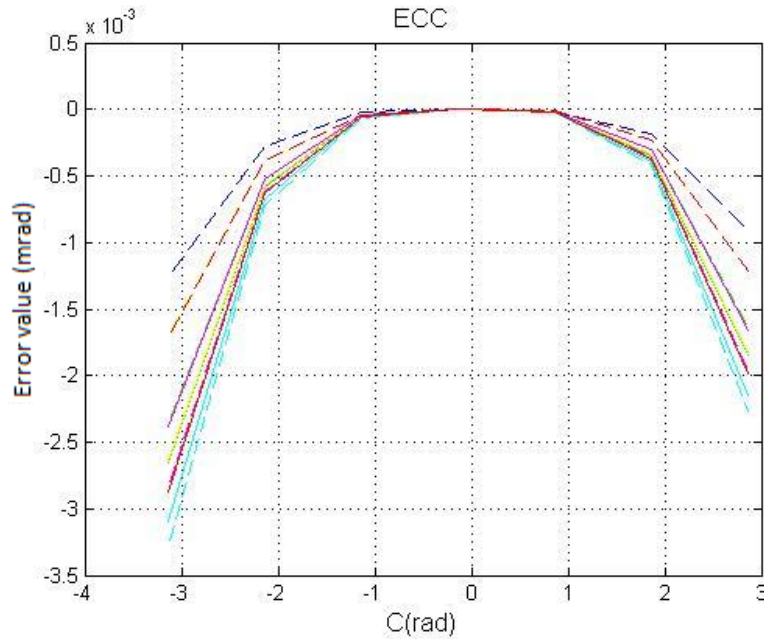


Figure 5-28: The polynomial graphs of the error ECC

There is no ECC error at C equal to zero. In other words, when the machine pallet doesn't rotate around its C axis, the ECC error is zero; but as soon as the pallet rotates around C, the ECC error will be created.

From color blue to color red, the error value generally increases by adding more load on the machine.

All the graphs have the same trend and quite symmetrical.

The maximum absolute error value is around 0.003 mm or 3  $\mu\text{m}$ .

The values of the errors for spindle (EXA, EYA, EZA, EAA, EBA and ECA) have been considered to be zero. It means that the spindle errors don't have any large effect on the machine tools geometry. In addition, the range of motion errors for EAY, EBY, ECY and EBZ are equal to zero. These error parameters have not been processed because of their small values and insignificant effect on the machine.

All the types of the errors have been analyzed. The maximum value belongs to the error EZB which was 0.5 mm and the minimum value belongs to the error EBC which was equal to 0.00005 mm. Most of the error types increased by adding more loading fixtures on the machine tools, although there were some errors which were unchanged or decreased by increasing weights. In general, by approaching to the zero point of the axis position (X, Y, Z, A, B and C), the value of the error decreased (in most cases the error value is going to be zero). Thus, it can be concluded that the value of the error was high in the maximum and minimum range of the position and was minimal around the zero point of the position.

## 5.2 Discrepancy of the errors

It is also possible to verify the range of the error versus weight and torque in 3D graph. This can help to see the performance of the machine in different loading conditions with respect to each error type. To this aim, the minimum and maximum values of each range of the motion errors were calculated and then sketched for different weights and torques. Equation 5-10 calculates the difference between the max and the min values of error EXX. The other error parameters follow the same equation. Figure 5-29 shows all the error types. In these figures, the colors are in the same order as the polynomial graphs (blue, cyan, green, yellow, magenta and red). Plotting the graphs has been done in Matlab with 'stem' function (Appendix I).

$$\Delta EXX_i = \max(EXX_i) - \min(EXX_i)$$

Where 'i' is the number of the test.

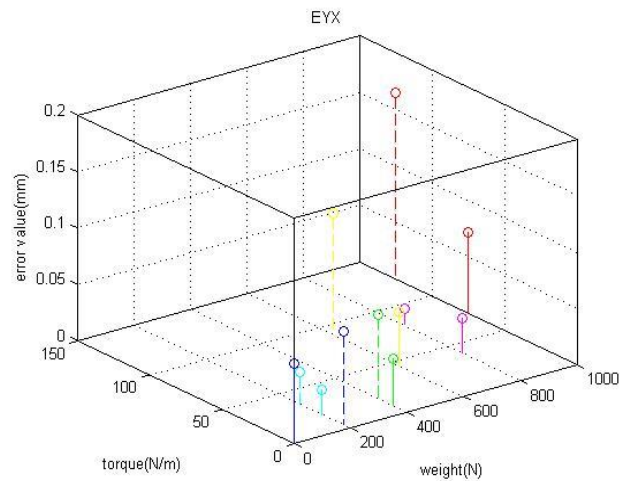
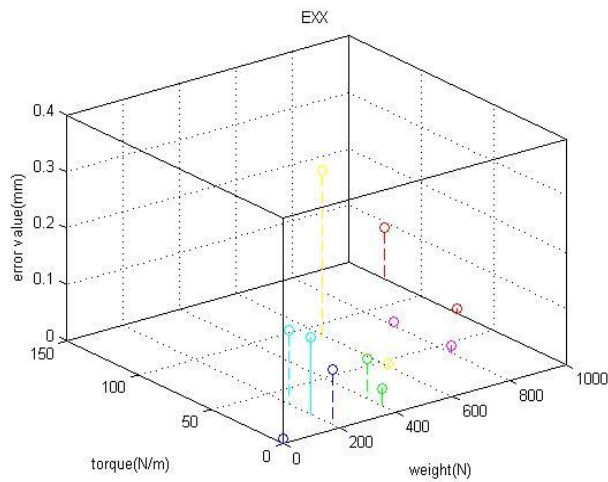
In these graphs, the discrepancy of each type of the error in different loading conditions is shown. It can be seen that the discrepancy of red and magenta bars was higher than the other bars indicating that the performance of the machine tool reduced by adding more loads on it. It is also

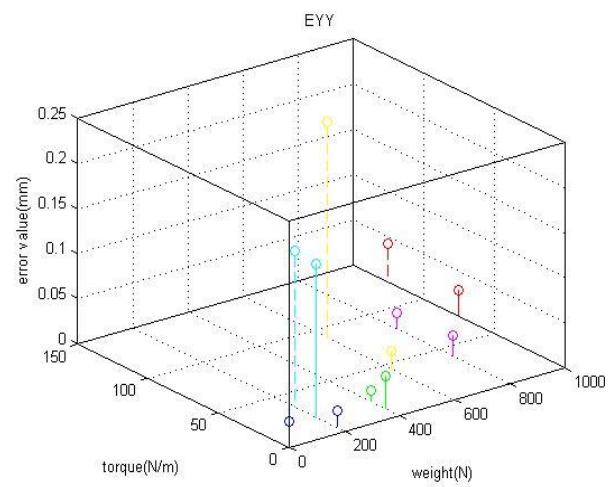
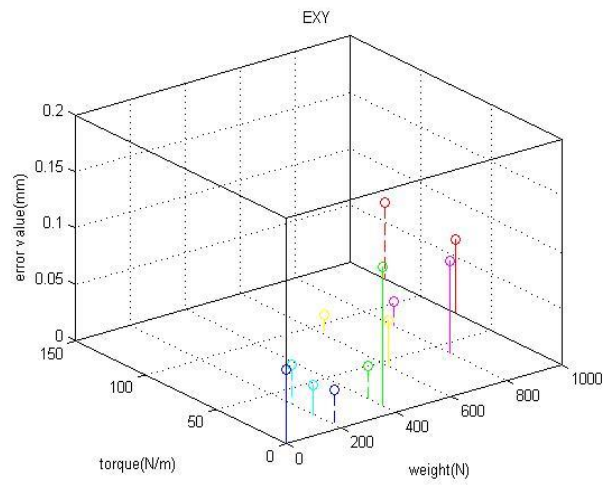
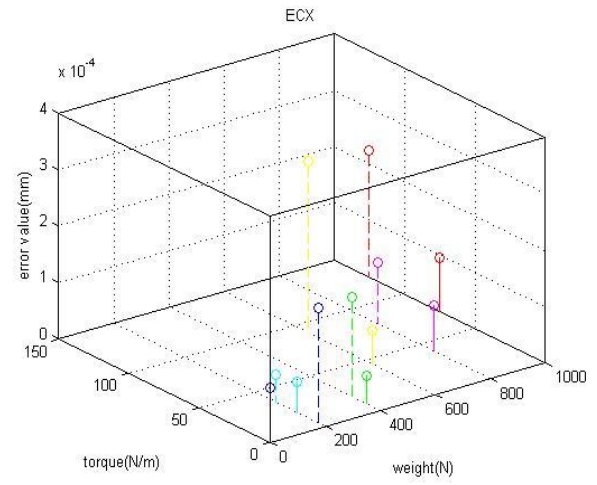
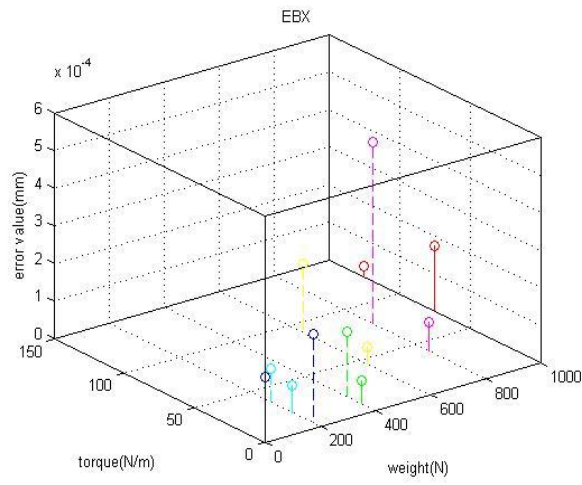
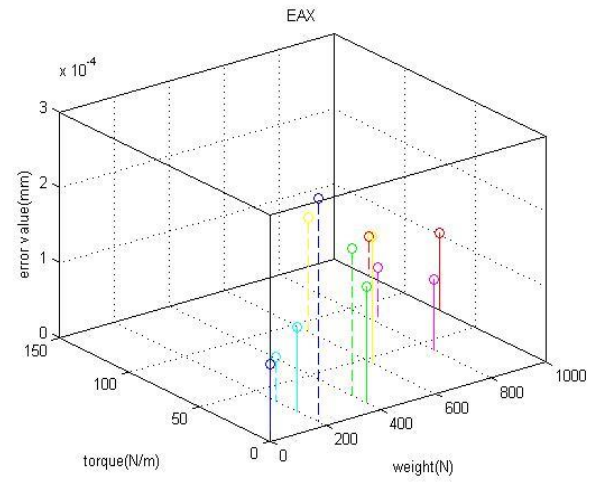
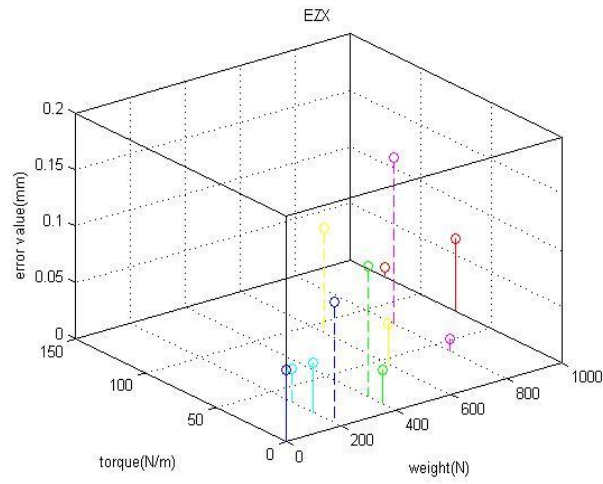


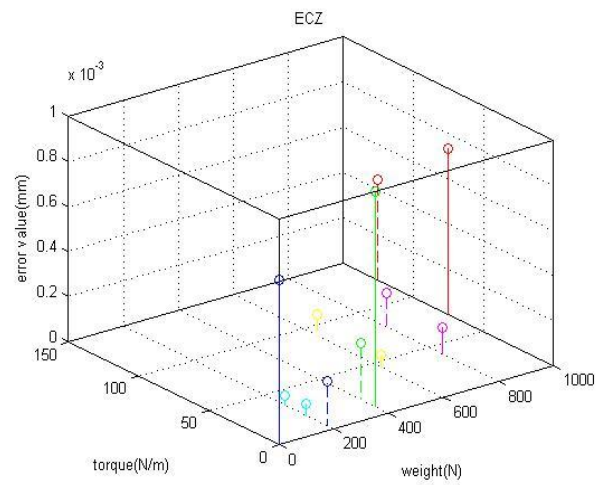
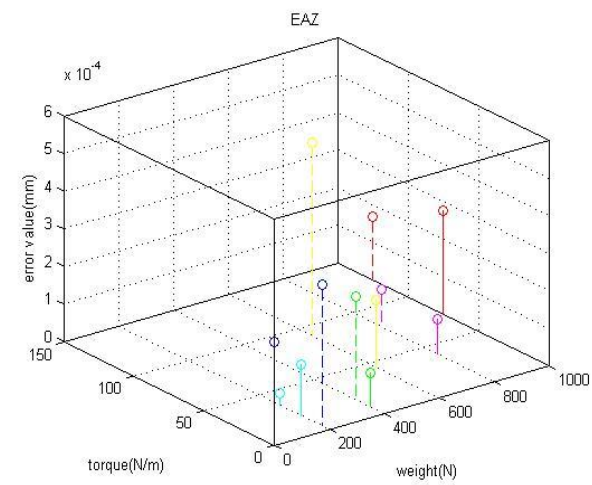
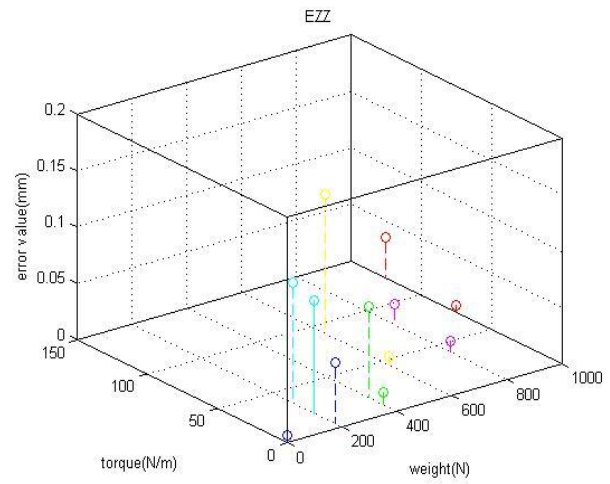
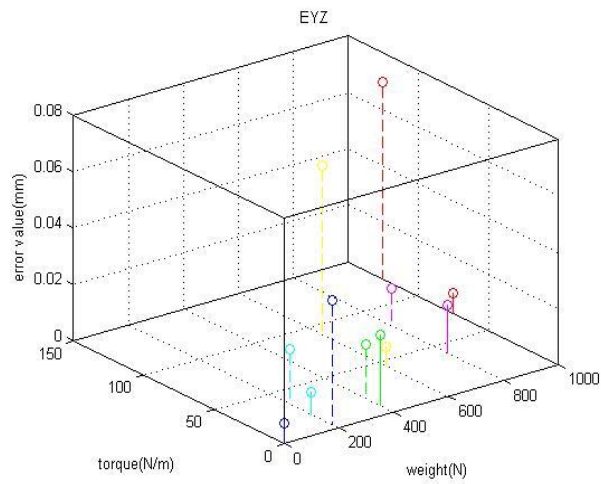
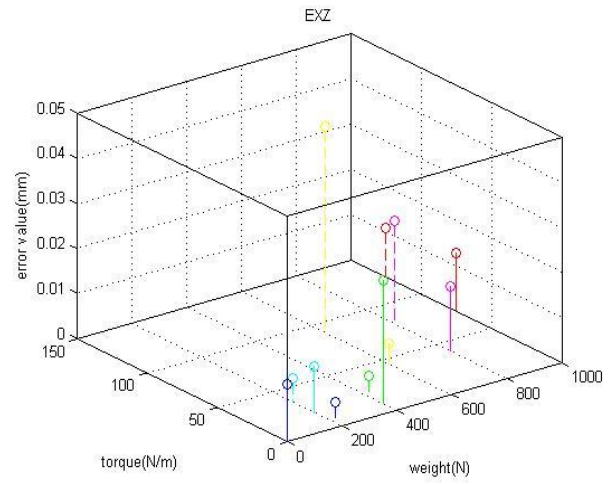
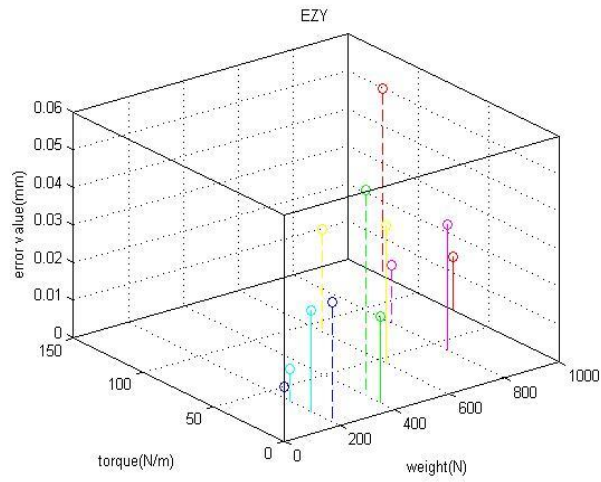
possible to compare only the values for same weight and different torques. In this case, the range of the error generally increased by

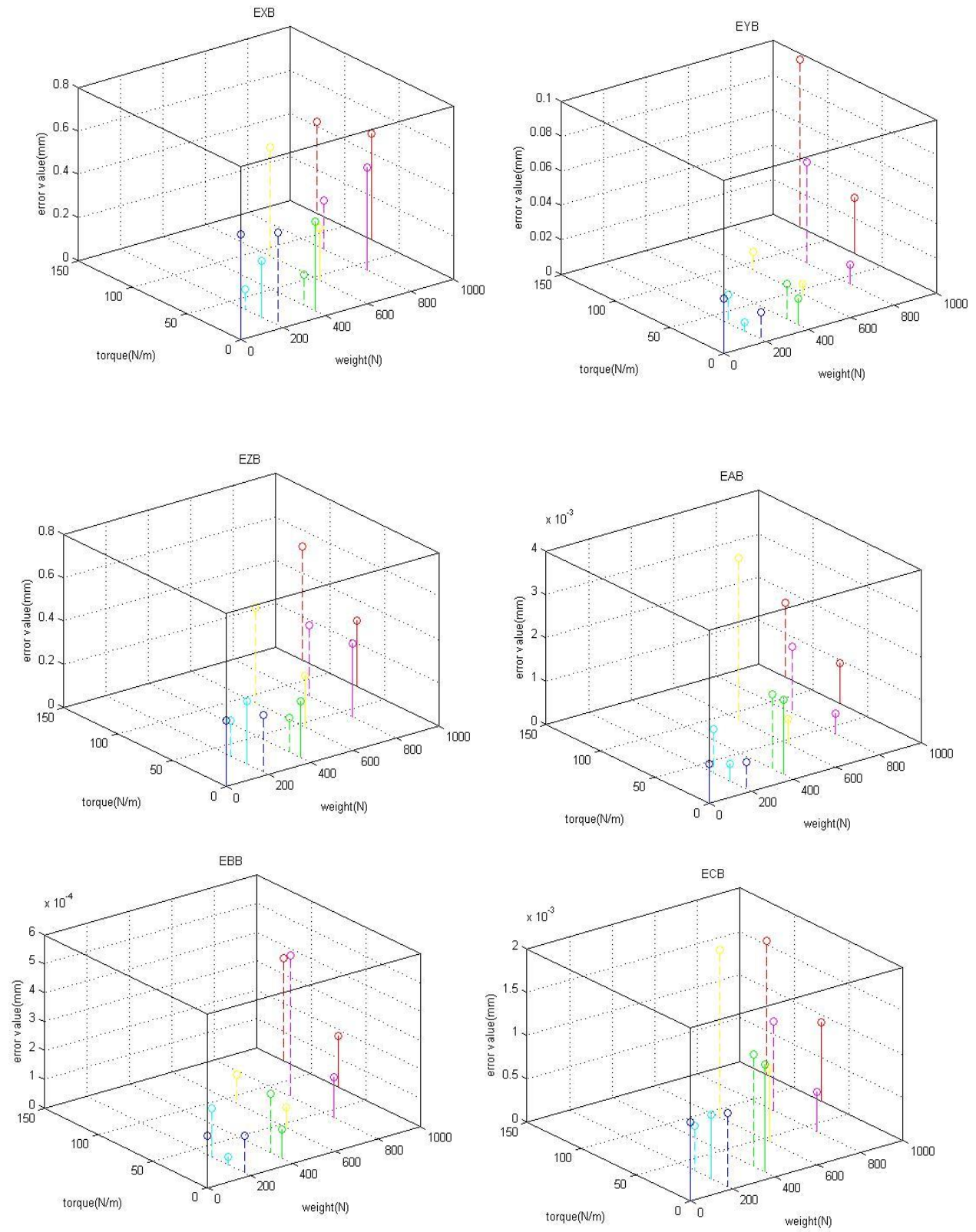
increasing the value of the torque. Therefore, torque had contributing effect in increasing the value of the errors.

As it is shown in figure 5-29 like in polynomial graphs in previous section, the maximum value of the error belongs to the EZB and the minimum value of the error belongs to the EBC. In addition, the value of the error increases for most errors by adding more loads on the machine (from color blue, minimum weight and torque, to color red, maximum weight and torque). In general, it could be concluded that the performance of the machine is more affected by increasing weight and torque values.











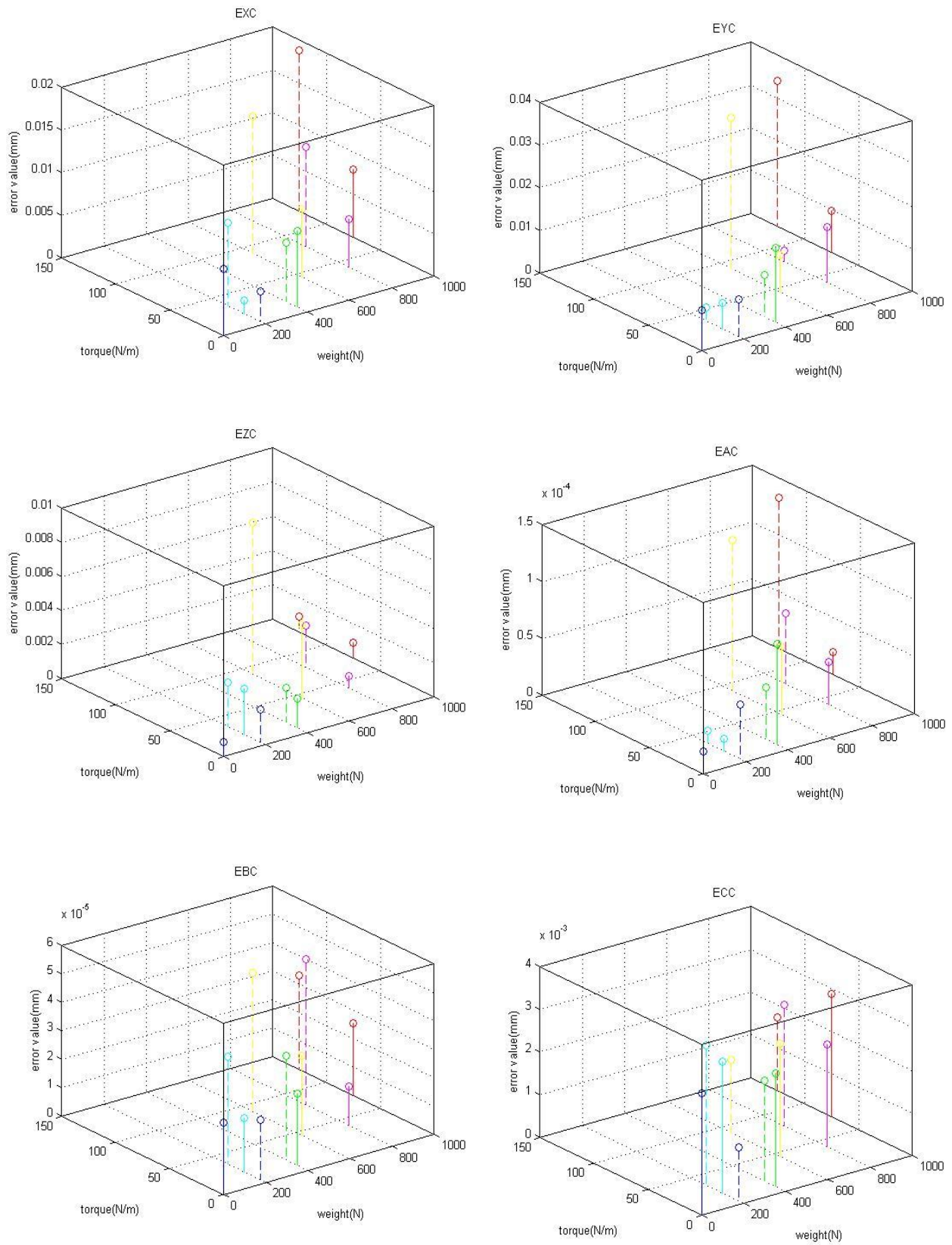


Figure 5-29: Discrepancy of the errors in different tests

### 5.3 Volumetric Errors

The volumetric error of the tool is the difference between the actual and the nominal position of the tool. The volumetric error of the workpiece has the same definition (difference between the actual position of the workpiece and the nominal position of the workpiece). The volumetric error of the machine tool is the difference of the tool volumetric error and the workpiece volumetric error.

$$\{F\},F\vec{\delta}_t = \{F\},F P_t^V - \{F\},F P_t^N \quad \text{The volumetric error of the tool}$$

Where  $\{F\},F P_t^V$  and  $\{F\},F P_t^N$  are the actual and the nominal position of the tool respectively which are projected in the foundation frame of the machine.

$$\{F\},F\vec{\delta}_w = \{F\},F P_w^V - \{F\},F P_w^N \quad \text{The volumetric error of the workpiece}$$

Where  $\{F\},F P_w^V$  and  $\{F\},F P_w^N$  are the actual and the nominal position of the workpiece respectively which are projected in the foundation frame of the machine.

$$\vec{\delta}_{O_w O_t} = \{F\},F\vec{\delta}_t - \{F\},F\vec{\delta}_w \quad \text{The volumetric error of the machine}$$

The volumetric error can be also written as a function of the errors. The relationship between the volumetric error and machine errors is defined by the Jacobian matrix which is shown in following equation:

$$VE = J \delta_p$$

Where ‘VE’ is the volumetric error, ‘J’ is the Jacobian matrix and ‘ $\delta_p$ ’ is the error parameters matrix.

All the equations for calculating the position of the tool, the position of the workpiece, the volumetric error matrix and the Jacobian matrix have been presented in appendix III and appendix IV respectively. according to the nominal and the exact modeling of the machine HU40T (the nominal and the exact kinematic chain have been shown in figure 3-5 and figure 3-6 in chapter 3 respectively.).

Figures 5-30 and 5-31 show the maximum and mean volumetric error for different tests. Each graph presents the volumetric error value before and after modeling. It is expected that the values after modeling are much smaller than the previous ones.

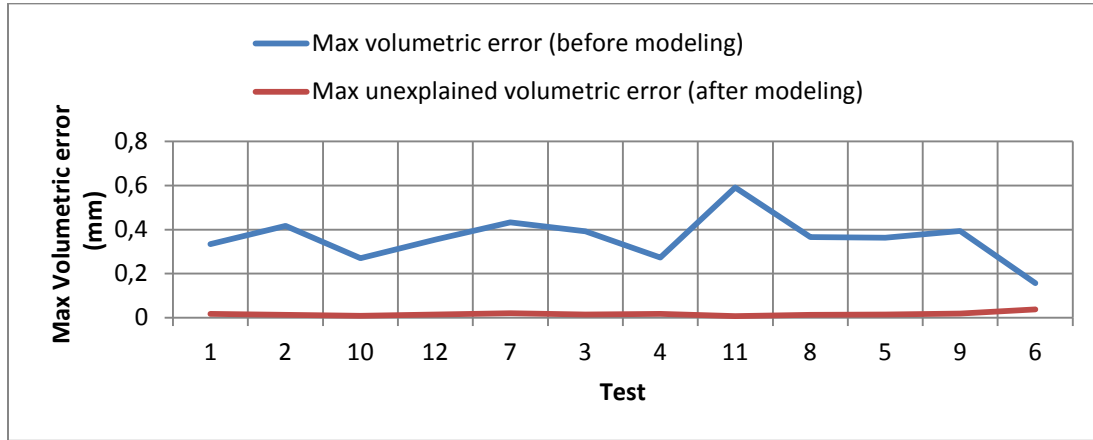


Figure 5-30: The maximum volumetric error for different tests

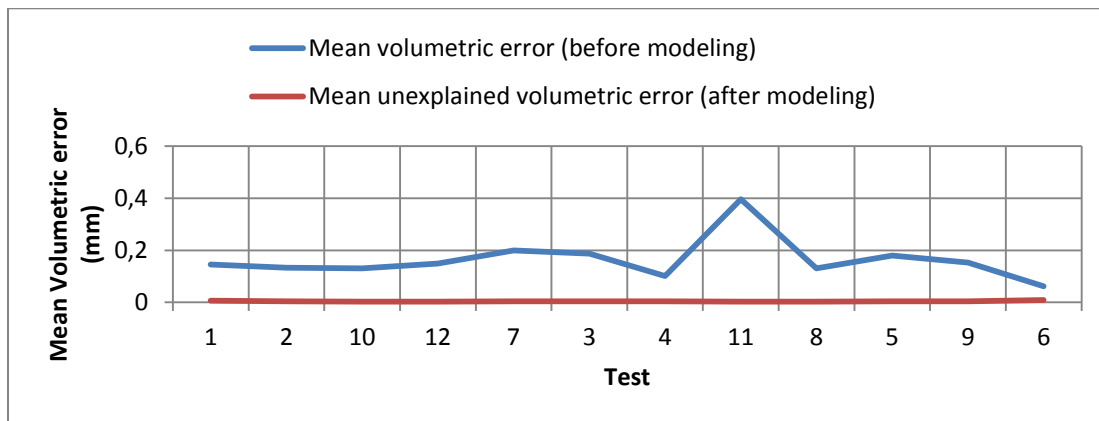


Figure 5-31: The mean volumetric error for different tests

The volumetric error before modeling presents the error value that come directly from the machine and the volumetric error after modeling presents the error value that the model (RUMBA) estimates and these values are close to zero in different loading conditions. Thus, the model can perfectly explain the loaded machine and estimate the value of the error coefficients with a high accuracy. In addition, as it is shown, the volumetric error is not affected significantly by increasing weight and torque. One reason of this can be due to the fact that the tool branch and the workpiece branch neutralize their errors and therefore, the difference of the volumetric errors is not too large.

## Chapter 6 Error modeling

In this chapter, we aimed to: (i) find a correlation between error coefficients, weights and torques, (ii) model all the estimated error coefficients as a function of weight, torque and the combination of weight and torque, and (iii) find the goodness of the fitting. The model then can be used to predict and calculate the value of each error coefficient and also each range of error without doing any other experimental test. The total number of the error coefficients are 180 (5 times 36 types of errors), although some of them are equal to zero. Table 6-1 shows all the possible error coefficients.

Table 6-1: All the possible error coefficients

EXX0	EYX0	EZX0	EAX0	EBX0	ECX0	EXY0	EYY0	EZY0	EAY0	EBY0	ECY0
EXX1	EYX1	EZX1	EAX1	EBX1	ECX1	EXY1	EYY1	EZY1	EAY1	EBY1	ECY1
EXX2	EYX2	EZX2	EAX2	EBX2	ECX2	EXY2	EYY2	EZY2	EAY2	EBY2	ECY2
EXX3	EYX3	EZX3	EAX3	EBX3	ECX3	EXY3	EYY3	EZY3	EAY3	EBY3	ECY3
EXX4	EYX4	EZX4	EAX4	EBX4	ECX4	EXY4	EYY4	EZY4	EAY4	EBY4	ECY4

EXZ0	EYZ0	EZZ0	EAZ0	EBZ0	ECZ0	EXA0	EYA0	EZA0	EAA0	EBA0	ECA0
EXZ1	EYZ1	EZZ1	EAZ1	EBZ1	ECZ1	EXA1	EYA1	EZA1	EAA1	EBA1	ECA1
EXZ2	EYZ2	EZZ2	EAZ2	EBZ2	ECZ2	EXA2	EYA2	EZA2	EAA2	EBA2	ECA2
EXZ3	EYZ3	EZZ3	EAZ3	EBZ3	ECZ3	EXA3	EYA3	EZA3	EAA3	EBA3	ECA3
EXZ4	EYZ4	EZZ4	EAZ4	EBZ4	ECZ4	EXA4	EYA4	EZA4	EAA4	EBA4	ECA4

EXB0	EYB0	EZB0	EAB0	EBB0	ECB0	EXC0	EYC0	EZC0	EAC0	EBC0	ECC0
EXB1	EYB1	EZB1	EAB1	EBB1	ECB1	EXC1	EYC1	EZC1	EAC1	EBC1	ECC1
EXB2	EYB2	EZB2	EAB2	EBB2	ECB2	EXC2	EYC2	EZC2	EAC2	EBC2	ECC2
EXB3	EYB3	EZB3	EAB3	EBB3	ECB3	EXC3	EYC3	EZC3	EAC3	EBC3	ECC3
EXB4	EYB4	EZB4	EAB4	EBB4	ECB4	EXC4	EYC4	EZC4	EAC4	EBC4	ECC4



After removing the null values, the number of error coefficients is equal to 83 as shown in table 6-2.

Table 6-2: The 83 estimated error coefficients

EXX2	EYX2	EAX0		ECX0	EXY0	EYY0					EAZ0	ECZ0
EXX3	EYX3	EAX1	EBX1	ECX1		EYY1		EXZ1		EZZ1	EAZ1	ECZ1
EZX2	EZX3	EAX2	EBX2	ECX2	EXY2	EYY2	EZY2	EXZ2	EYZ2	EZZ2	EAZ2	ECZ2
		EAX3	EBX3	ECX3	EXY3	EYY3	EZY3	EXZ3	EYZ3	EZZ3	EAZ3	ECZ3
EXB0			EAB0	EBB0	ECB1	EXC1	EYC1	EZC1	EAC1	EBC1		
EXB1	EYB1	EZB1	EAB1	EBB1	ECB2	EXC2	EYC2	EZC2	EAC2	EBC2	ECC1	
EXB2	EYB2	EZB2	EAB2	EBB2	ECB3	EXC3	EYC3	EZC3	EAC3	EBC3	ECC2	
EXB3	EYB3	EZB3	EAB3	EBB3							ECC3	
				EBB4							ECC4	

## 6.1 Correlation

Dependency between two set of data is defined as a relationship between their values. Correlation is a statistical relationship which shows the value of the dependency. For instance, electrical sources produce less power on a mild day and this is based on the correlation between electricity and weather. “In this example there is a causal relationship, because extreme weather causes people to use more electricity for heating or cooling; however, statistical dependence is not sufficient to demonstrate the presence of such a causal relationship” [26,27].

There are different types of correlation coefficient which measure the degree of correlation. The common correlation coefficient is called ‘Pearson Correlation Coefficient’ which is suitable for showing the linear relationship between two variables. It is obtained by dividing the covariance of the two variables by the product of their standard deviations:

$$Corr(X,Y) = \frac{cov(X,Y)}{\sigma_x \sigma_y} \quad (6-1)$$



In the input, all the values of the error coefficient were close to zero and Matlab considers these values as null. Thus, most of the elements of the correlation matrix is shown as NaN (Not-a-number). A NaN is obtained as a result of mathematically undefined operations like 0.0/0.0 and inf-inf. For solving this problem, we normalized each column of the input matrix by dividing the elements of the column by the norm of the column. The norm of a matrix is a scalar that gives some measure of the magnitude of the elements of the matrix. The norm of a matrix 'X' can be calculated by following equation:

$$X = [X_1, X_2, \dots, X_n]$$

$$Norm(X) = \sqrt{X_1^2 + X_2^2 + \dots + X_n^2} \quad (6-4)$$

After running the Matlab program, the correlation matrix was created (Appendix I). This matrix gave us all the correlation values between the error coefficients and weight and torque. The last two columns and the last two rows represent the correlation of the error coefficients and weight and torque. All the elements of the correlation matrix have been presented in appendix V.

### 6.1.1 Correlation between the error coefficients

The error coefficients are defined as a degree of polynomial equation for each range of motion error. It is expected that there is a correlation between the zero degree and the second degree of each range of error (ex. EYY0 and EYY2) and also between first degree and third degree of the errors (ex. EYY1 and EYY3). The reason was that these coefficients were linked together two by two which was confirmed by the results (APPENDIX V). The dependency between EYY0 and EYY2 was equal to -0.748 and the dependency between EYY1 and EYY3 was equal to -0.977. The negative sign means that these two parameters are inversely correlated. There was a similar relation between the other group of degrees of errors (0 and 2, 1 and 3) regardless of the error data.

### 6.1.2 Correlation between the error coefficients and weight and torque

Table 6-3: Correlation matrix for weight and torque

The last two columns of the correlation matrix belong to the values of dependency between the error coefficients and the two parameters weight and torque. With these data, it is possible to verify which error is most correlated to the cited parameters. EYY0 and EZZ2 are most correlated to the weight and torque respectively which are equal to 0.505 and 0.685. EZZ1 and EYX3 are correlated to W and T but reversely which are equal to -0.573 and -0.746 respectively. On the other side, EBC3 and EBX3 have the smallest dependency with the weight and torque which are equal to 0.002 and -0.030 respectively. Weight and torque are correlated with the dependency 0.829. The reason of the correlation between these two parameters is that weight and torque are linked together and torque is actually a function of weight. The error coefficient EZB1 is also correlated to torque with a correlation -0.739. This correlation can explain the large value of the motion error EZB in polynomial graphs.

	W	T
EXX2	0,048	0,399
EXX3	0,053	0,436
EYX2	-0,303	-0,425
EYX3	-0,380	-0,746
EZX2	-0,362	-0,439
EZX3	-0,366	-0,267
EAX0	-0,153	-0,284
EAX1	-0,366	-0,319
EAX2	-0,228	-0,126
EAX3	0,243	0,174
EBX1	0,488	0,677
EBX2	0,320	0,262
EBX3	0,054	-0,030
ECX0	0,217	0,551
ECX1	-0,180	-0,425
ECX2	-0,324	-0,633
ECX3	-0,043	-0,049
EXY0	-0,369	-0,661
EXY2	0,297	0,095
EXY3	-0,039	0,222
EYY0	0,505	0,475
EYY1	-0,220	0,181
EYY2	-0,211	0,042
EYY3	0,282	-0,120
EZY2	0,301	0,408
EZY3	-0,044	-0,375
EXZ1	0,258	0,054
EXZ2	-0,334	-0,170
EXZ3	0,325	0,150
EYZ2	0,140	0,410
EYZ3	0,098	-0,059
EZZ1	-0,573	-0,527
EZZ2	0,275	0,685
EZZ3	0,027	-0,394
EAZ0	-0,260	-0,597
EAZ1	0,143	0,516
EAZ2	-0,168	-0,584
EAZ3	0,238	0,662
ECZ0	-0,219	-0,417
ECZ1	0,145	0,310
ECZ2	-0,096	-0,205
ECZ3	0,055	0,085

	W	T
EXB0	-0,436	-0,743
EXB1	0,246	0,524
EXB2	0,362	0,675
EXB3	-0,265	-0,503
EYB1	-0,226	-0,197
EYB2	-0,284	-0,480
EYB3	0,277	0,324
EZB1	-0,437	-0,739
EZB2	0,237	0,524
EZB3	0,204	0,469
EAB0	0,091	0,123
EAB1	0,211	0,553
EAB2	-0,109	-0,144
EAB3	-0,125	-0,464
EBB0	-0,329	-0,367
EBB1	0,090	0,414
EBB2	0,098	-0,235
EBB3	-0,192	-0,461
EBB4	0,456	0,639
ECB1	0,017	0,030
ECB2	0,212	0,545
ECB3	0,002	-0,082
EXC1	0,281	0,638
EXC2	-0,154	-0,496
EXC3	-0,290	-0,649
EYC1	0,390	0,572
EYC2	-0,131	-0,407
EYC3	-0,393	-0,565
EZC1	0,078	0,371
EZC2	0,004	-0,187
EZC3	-0,092	-0,368
EAC1	0,273	0,405
EAC2	0,139	0,091
EAC3	-0,272	-0,395
EBC1	-0,008	-0,165
EBC2	0,205	0,559
EBC3	0,002	0,154
ECC1	0,273	0,489
ECC2	0,201	0,038
ECC3	-0,139	-0,370
ECC4	0,182	0,247
W	1,000	0,829
T	0,829	1,000

There is also a relation between the number of degrees:

- If the 2nd degree and the 3rd degree are correlated to weight or torque, the 0th degree and the 1st degree will have to be correlated to these parameters as well. This does not mean that these degrees are necessarily correlated to  $W$  and  $T$ . It can be due to the correlation of the 2nd degree and the 3rd degree with  $W$  and  $T$ .
- If the 0th degree and the 1st degree are correlated to weight or torque, the 2nd and the 3rd degree do not necessarily correlate and even, if they are correlated, it is as a result of the weight or torque.

## 6.2 Curve fitting

After finding the correlation matrix between the error coefficients and weight and torque, each error coefficient can be modeled as a function of weight, torque and a combination of weight and torque. To this aim, a model was fitted to the data (estimation result for each error coefficient) to achieve the modeling equation. Thus, each error coefficient was presented by the following equation:

$$E_{n,i} = F(W, T) \quad \text{where 'i' can be X, Y, Z, A, B or C}$$

$n$  is the degree of the error coefficient which can be 0, 1, 2, 3 or 4 (number 4 is the backlash coefficient).  $W$  and  $T$  are weight and torque respectively.

The Curve fitting methodology is to find the best fit between the set of data. This curve shows the trend of the data and predicts the unavailable data points. In other words, the curve fitting modeling defines the relationships between different variables [29,30].

The process of curve fitting was programmed in Matlab and the graphs were plotted. There are 83 graphs showing the model of the error coefficient with respect to the weight and torque.

This section has 3 parts:

1. Model the error coefficients as a function of only weight
2. Model the error coefficients as a function of only torque
3. Model the error coefficients as a function of both weight and torque

### 6.2.1 Model the error coefficients as a function of only weight

The 2D graph of each error coefficient versus weight shows the variation of the error coefficient in different loading conditions. Figure 6-1 shows the fitted curve for some different error coefficient.

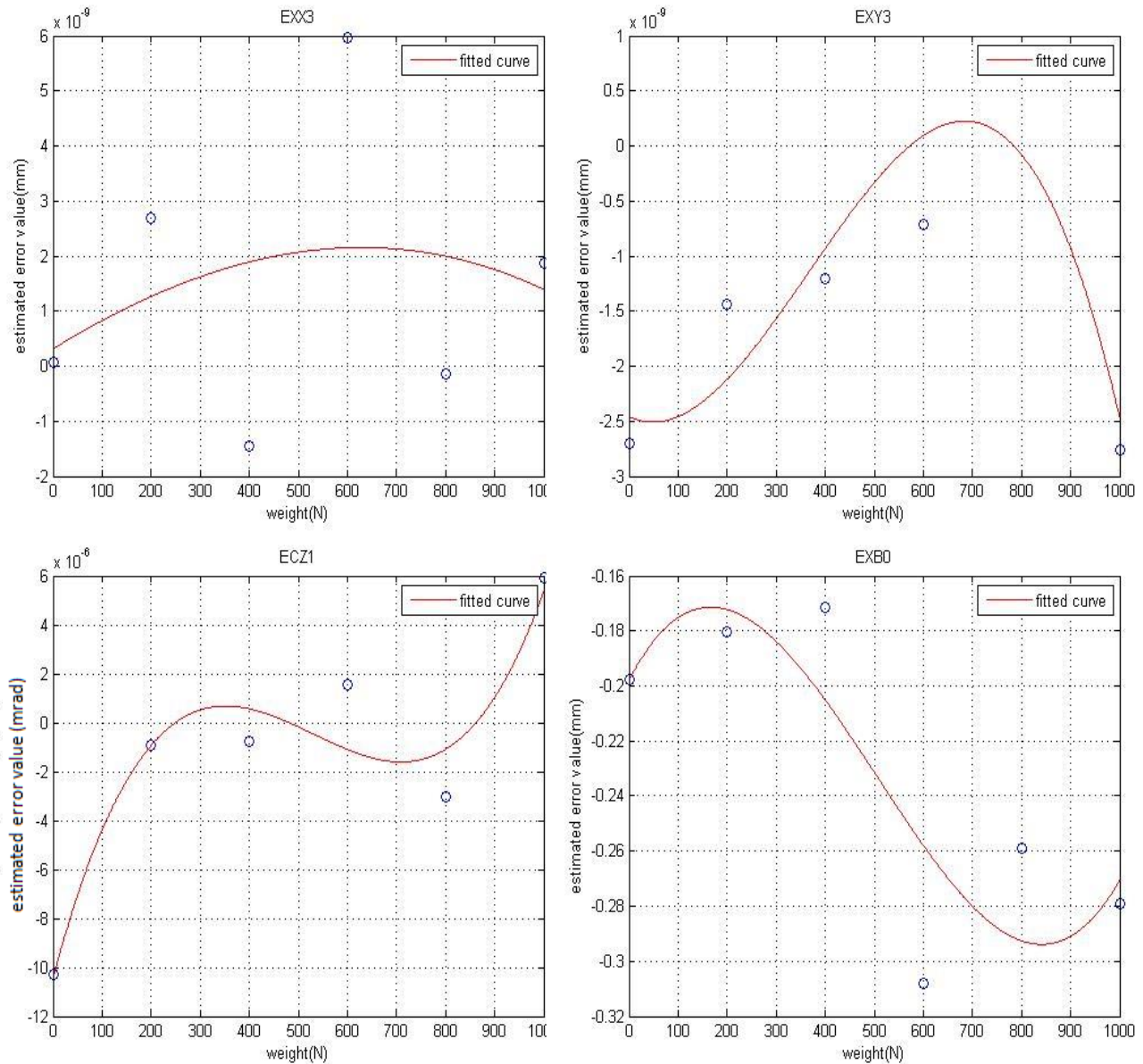


Figure 6-1: The error coefficients modeling as a function of weight

The degree of the polynomials for each error coefficient can be randomly defined considering some restrictions. The number of the points is equal to 6. Thus, if we choose the 5<sup>th</sup> degree for the curve fitting, the graph will pass all the points and this modeling is not valuable. Therefore, the 3<sup>rd</sup> degree of the polynomial was chosen to have a meaningful modeling and low residual (residual analysis will be explained in section 6.3).

The original formula of the error coefficient model is defined by following equation:

$$s(W) = p1 \times W^3 + p2 \times W^2 + p3 \times W + p4 \quad (6-5)$$

which 's(W)' is the error coefficient and 'W' is weight. The following equation presents the error coefficients modeling in matrix format:

$$\begin{bmatrix} s_1(W) \\ s_2(W) \\ \vdots \\ s_{82}(W) \\ s_{83}(W) \end{bmatrix}_{83 \times 1} = [PA]_{83 \times 4} \times \begin{bmatrix} W^3 \\ W^2 \\ W^1 \\ 1 \end{bmatrix}_{4 \times 1} \quad (6-6)$$

The curve fitting method has been done in Matlab with function 'fit' (Appendix I).

The elements of the matrix 'PA' for each error coefficient equation have been shown in appendix VI-table F-1.

### 6.2.2 Model the error coefficients as a function of only torque

This time, the error coefficients are defined as a function of torque. Thus, they can be estimated by any values of the torque. Some fitted graphs are shown in figure 6-2.

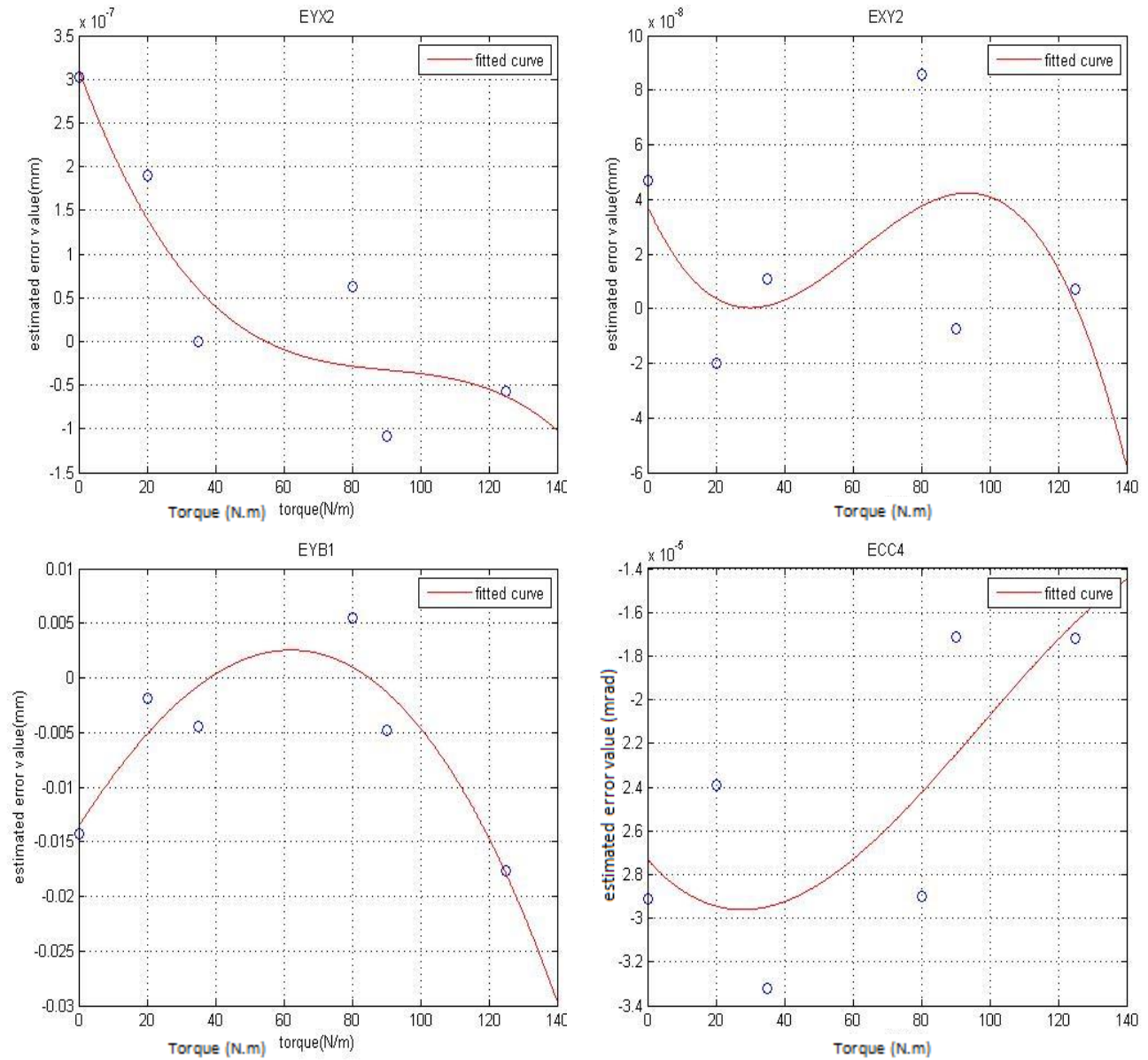


Figure 6-2: The error coefficients modeling as a function of torque



In this modeling, the number of points is also equal to six and the chosen degree of the polynomial curve fitting is 3 (the optimized number for the degree to have optimized fitted and low residual value).

The original formula of the error coefficient model is defined by following equation:

$$e(T) = p_1 \times T^3 + p_2 \times T^2 + p_3 \times T + p_4 \quad (6-7)$$

which 'e(x)' is the error coefficient and 'y' is torque. Thus, we can represent all the coefficients in a matrix format:

$$\begin{bmatrix} e_1(T) \\ e_2(T) \\ \vdots \\ e_{82}(T) \\ e_{83}(T) \end{bmatrix}_{83 \times 1} = [PB]_{83 \times 4} \times \begin{bmatrix} T^3 \\ T^2 \\ T^1 \\ 1 \end{bmatrix}_{4 \times 1} \quad (6-8)$$

The elements of the matrix 'PB' for each error coefficient equation have been shown in appendix VI-table F-2.

### 6.2.3 Model the error coefficient as a function of both weight and torque

In the third part, the error coefficients are defined as a function of weight and torque simultaneously. This model is more precise comparing to previous models, because, as mentioned before, the two parameters, weight and torque, are linked together and they affect each other by changing their values. Thus, with this model, we are able to see the error coefficient changes versus weight and torque together to verify and estimate the error value with any values of weight and torque at the same time. Therefore, there is a 3D graph for each error coefficient that it's X axis shows weight, Y axis shows torque and Z axis shows the error value. In this 3D graph, there is a surface to represent the model of the error coefficient versus weight and torque. Figure 6-3 shows some graphs for different error coefficients. The rest are shown in appendix VII.

The original formula of the mentioned model is defined in following equation:

$$E(W,T) = p_{00} + p_{10} \times W + p_{01} \times T + p_{20} \times W^2 + p_{11} \times W \times T + p_{02} \times T^2 + p_{30} \times W^3 + p_{21} \times W^2 \times T + p_{12} \times W \times T^2 + p_{03} \times T^3 \quad (6-9)$$

Which 'E(x,y)' is the error coefficient, 'x' is weight, 'y' is torque and 'p' are the coefficients.

The cited formula can be written as a matrix format:

$$\begin{bmatrix} E_1(W,T) \\ E_2(W,T) \\ \vdots \\ E_{82}(W,T) \\ E_{83}(W,T) \end{bmatrix}_{83 \times 1} = [P]_{83 \times 10} \times \begin{bmatrix} 1 \\ W^1 \\ T^1 \\ W^2 \\ WT \\ T^2 \\ W^3 \\ W^2T \\ WT^2 \\ T^3 \end{bmatrix}_{10 \times 1} \quad (6-10)$$

The elements of the matrix 'P' for each error coefficient equation have been shown in appendix VI-table F-3.

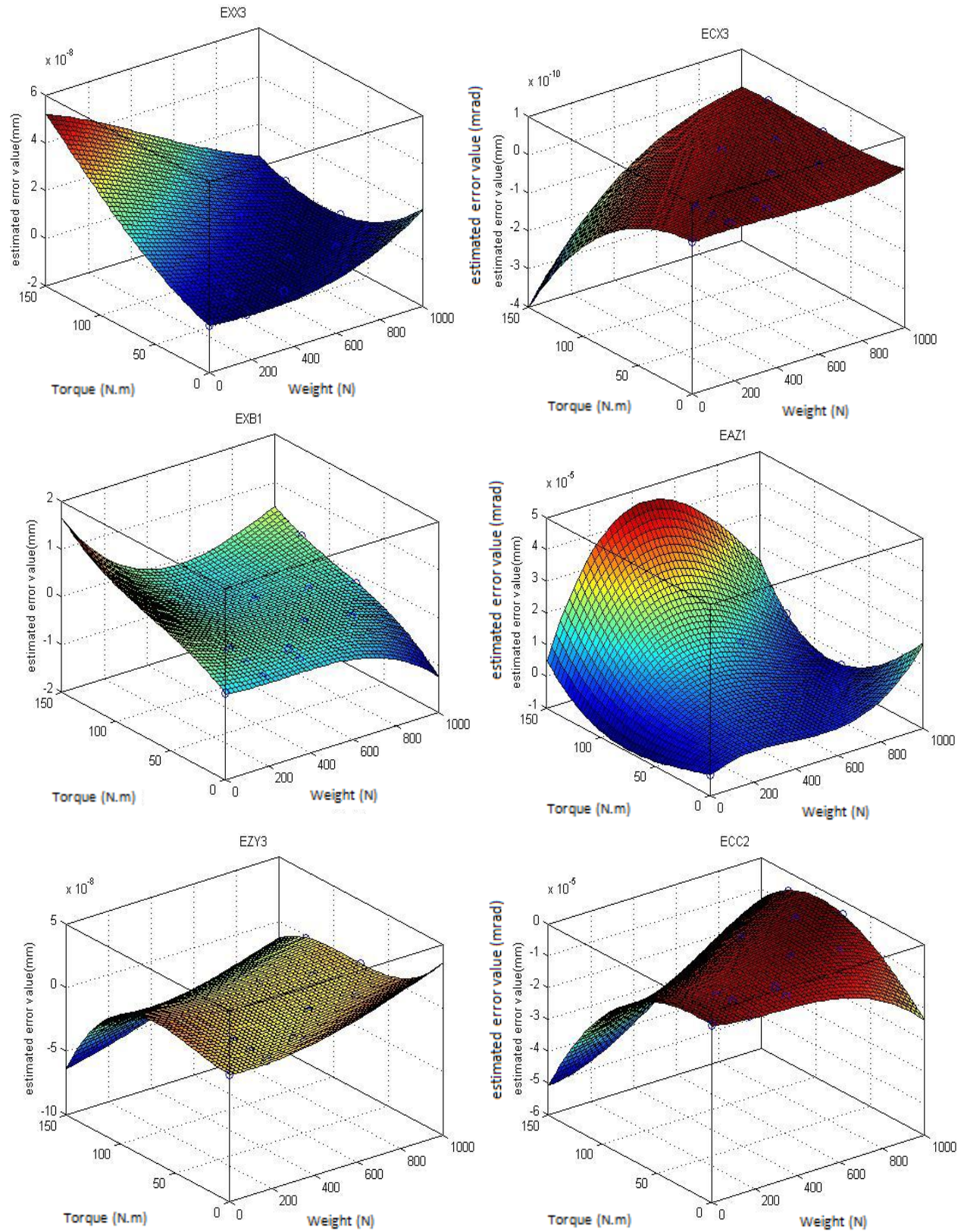


Figure 6-3: The error coefficients modeling as a function of weight and torque

### 6.3 Goodness-of-Fit Statistics (residual analysis)

After curve fitting, the goodness-of-fit is used to show how well the curve is fitted to the data set. One way to see the goodness of fit is analyzing the residual of the model. Residual analysis shows the difference between the observed values and the expected values from the model. The difference between the estimated error coefficient values and the values of curve fitting model is defined as a residual [31,32].

The result of goodness of fit is presented by different parameters such as ‘SSE’, ‘R-square’, ‘Adjusted R-square’, ‘RMSE’. R-square is the most common parameter for defining the lack of fit. This parameter is always between zero and one. The curve fitting is good when the R-square is close to one and the curve fitting is inaccurate when the value of the R-square is close to zero. The residual analysis can be explained with following simple example:

$X=1:10$ ;  $Y=\sin(X)$ ;

#### Fit 1:

In the first model, the cubic polynomial is used to model the data set. After processing the curve fitting method, the results have been achieved. Figure 6-4 shows the data fits and residuals for fit 1.

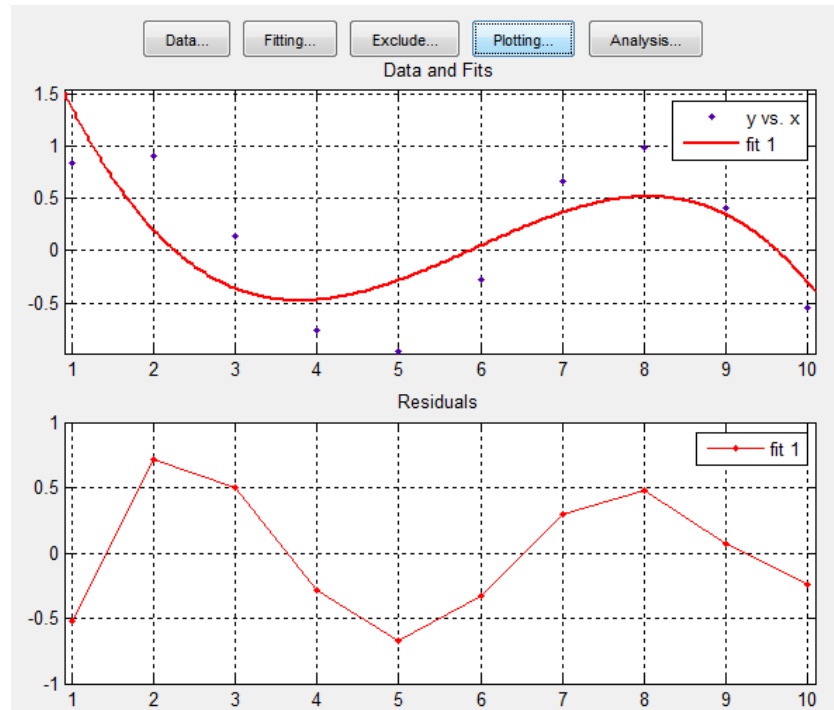


Figure 6-4: The data fit and the residual graph for Fit 1

$$f(x) = p1 \times x^3 + p2 \times x^2 + p3 \times x + p4$$

$$p4 = 3.295$$

Coefficients (with 95% confidence bounds):

Goodness of fit:

$$p1 = -0.02582$$

SSE: 2.05

$$p2 = 0.4583$$

**R-square: 0.5731**

$$p3 = -2.362$$

Adjusted R-square: 0.360

RMSE: 0.5846

As it is shown in residual graph, the goodness of fit is not acceptable (the value of the R-square is equal to 0.5731) and the value of the residual was quite big in relative to the data values. Thus, the higher degree for the polynomial function was chosen.

### Fit 2:

In the second fit, the 4<sup>th</sup> degree of the polynomial was used to model the data set. Figure 6-5 shows the data fits and residuals for fit 2.

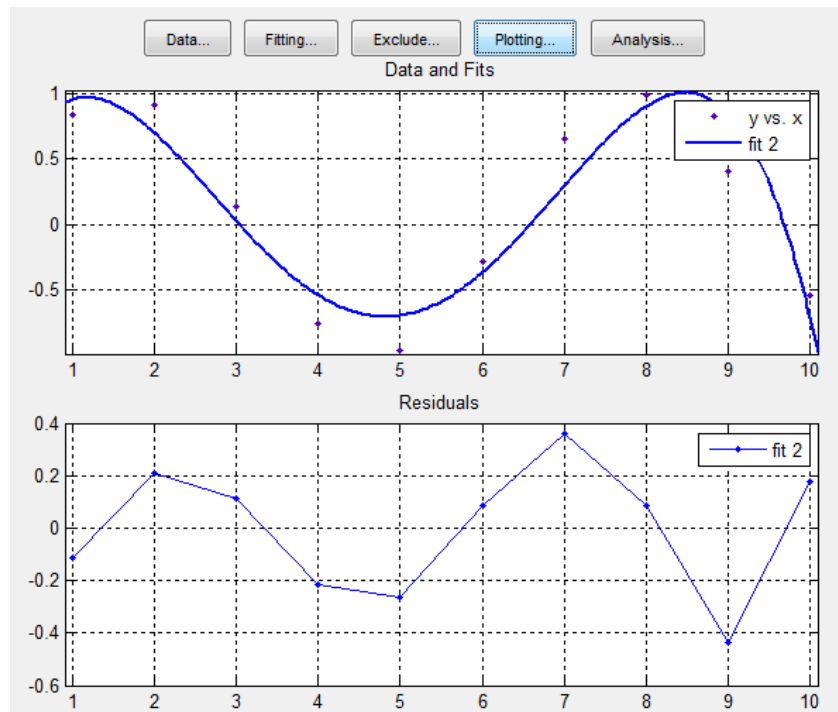


Figure 6-5: The data fit and the residual graph for Fit 2

Linear model Poly4:

$$p4 = 1.838$$

$$f(x) = p1 \times x^4 + p2 \times x^3 + p3 \times x^2 + p4 \times x + p5$$

$$p5 = 0.01862$$

Coefficients (with 95% confidence bounds):

Goodness of fit:

SSE: 0.5491

$$p1 = -0.009546$$

**R-square: 0.8857**

$$p2 = 0.1842$$

Adjusted R-square: 0.794

$$p3 = -1.079$$

RMSE: 0.3314

In the second fit, R-square was higher than the value in the first fit and the residual was smaller. However, higher degree was examined.

### Fit 3:

In the third fit, the 5<sup>th</sup> degree of polynomial was used to model the cited data set.

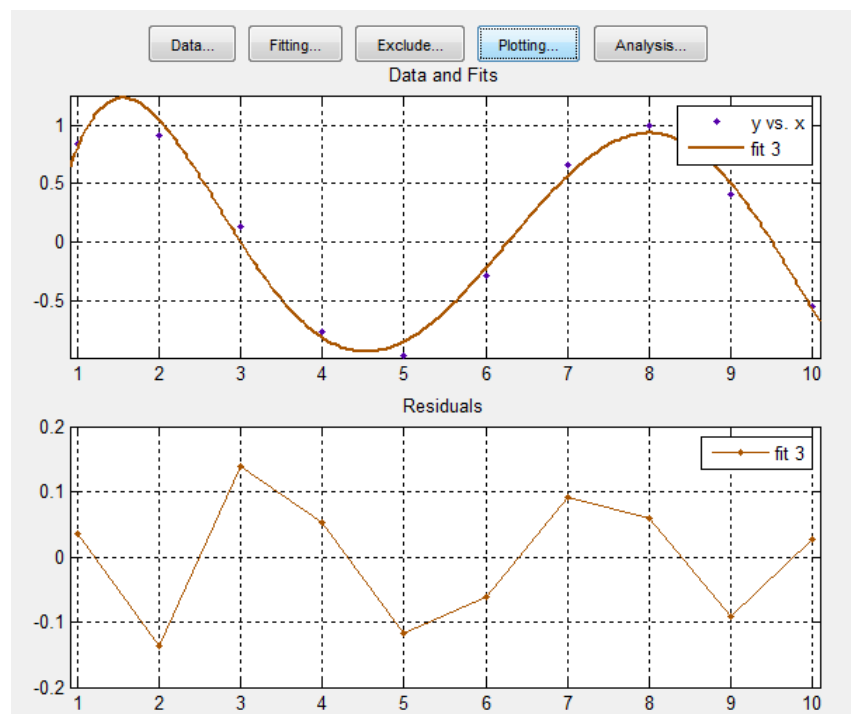


Figure 6-6: The data fit and the residual graph for Fit 3

Linear model Poly5:

$$p5 = 7.618$$

$$f(x) = p1 \times x^5 + p2 \times x^4 + p3 \times x^3 + p4 \times x^2 + p5 \times x + p6$$

$$p6 = -3.488$$

Coefficients (with 95% confidence bounds):

Goodness of fit:

SSE: 0.0802

$$p1 = 0.002452$$

**R-square: 0.9833**

$$p2 = -0.07697$$

Adjusted R-square: 0.962

$$p3 = 0.8625$$

RMSE: 0.1416

$$p4 = -4.113$$

In this fit, the value of R-square was equal to 0.9833 which was good enough for a model of curve fitting.

It can be concluded that the goodness of fit improves by increasing the number of the coefficients of the model. In this project, because there was no idea about the best degree for each error coefficient, the optimized fit (with minimum degree of polynomial and high R-square) was selected. So, the optimized degree of the polynomial functions for the error modeling was equal to 3.

The residual analysis has been done for all the models (as only a function of weight, function of torque and function of weight and torque). The value of the R-square for all the fitted curves are over than 0.7 and this is quite acceptable for the modeling. In Figure 6-7, some of the error modeling and their residual analysis as different functions have been shown.

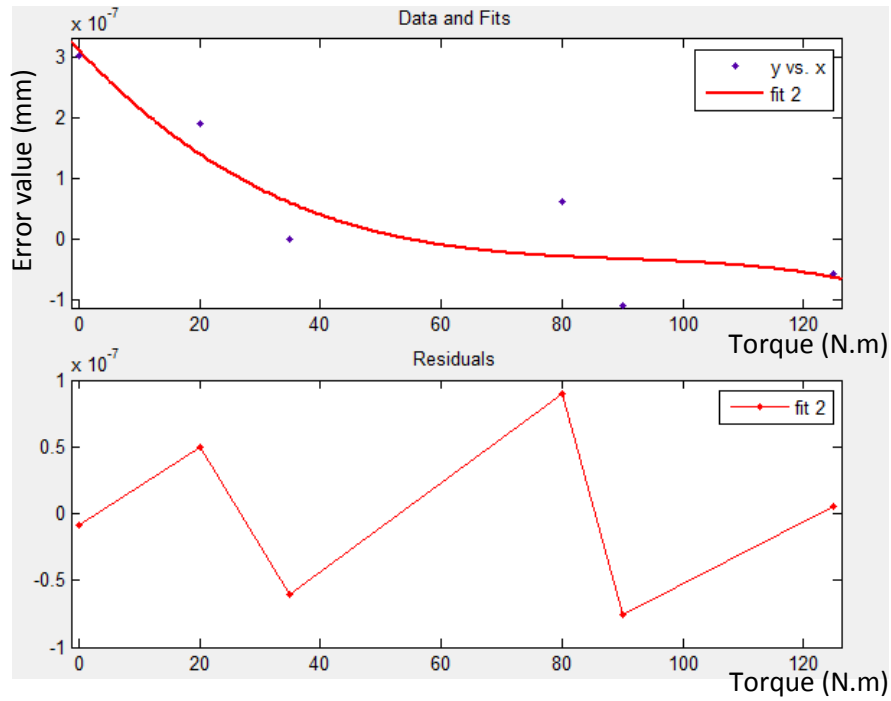


Figure 6-7-a: The residual analysis for error EYX2 as a function of torque with R-square 0.834

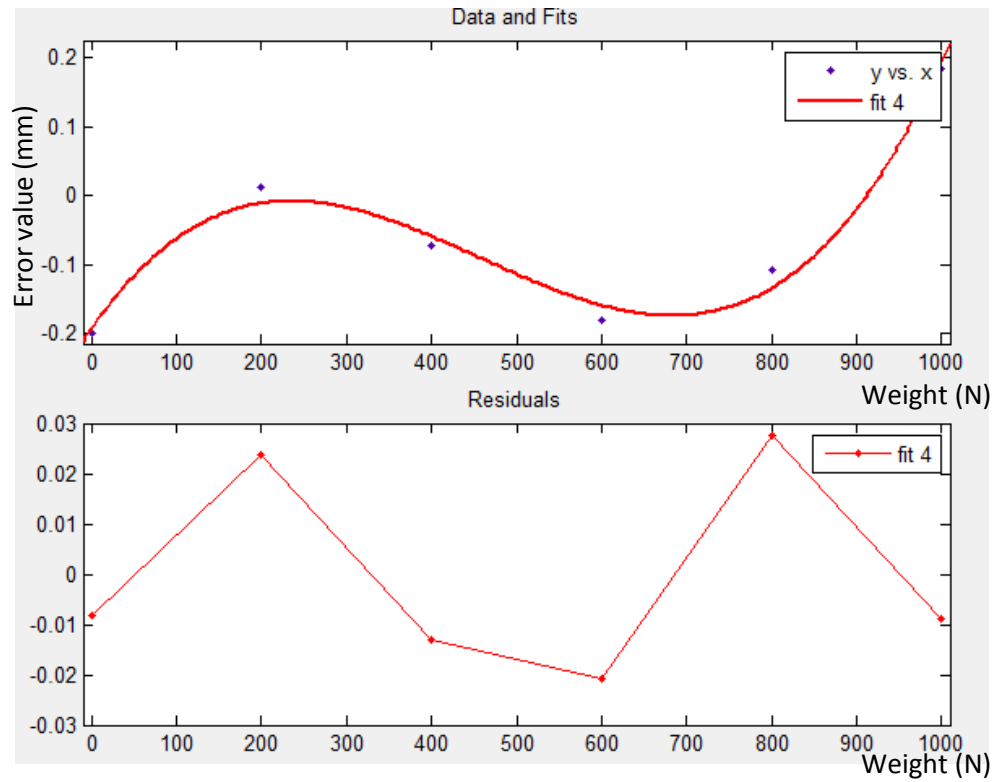


Figure 6-7-b: The residual analysis for error EXB1 as a function of weight with R-square 0.979



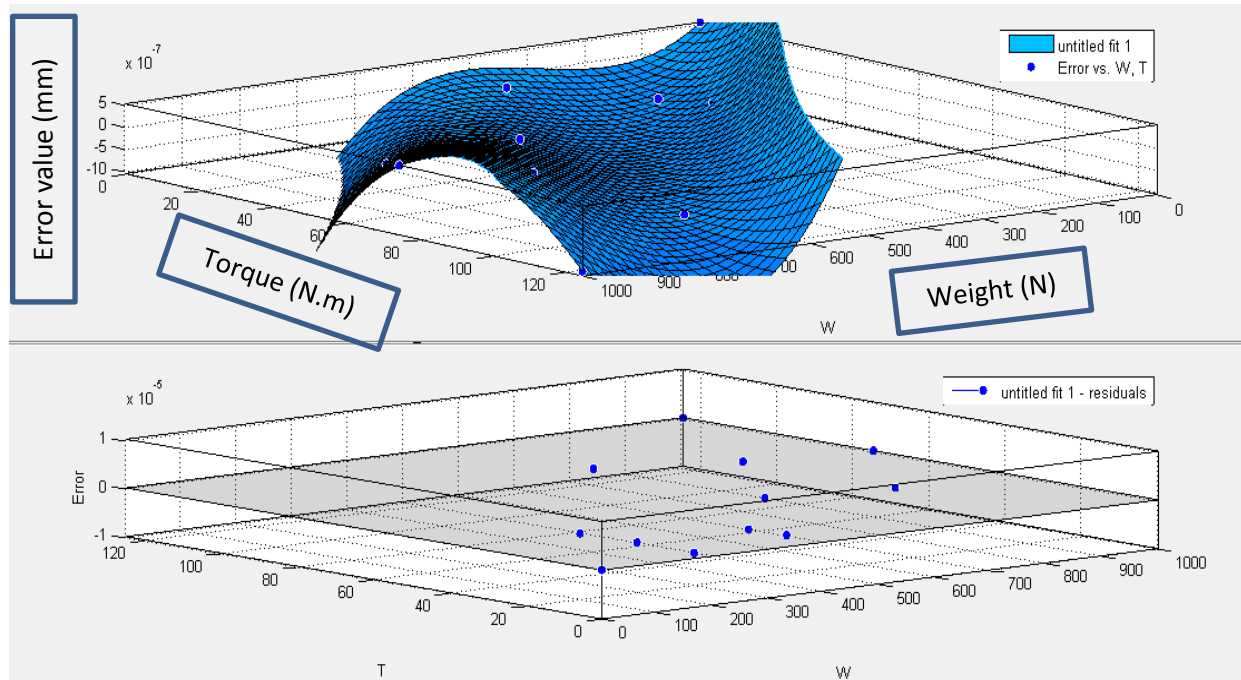


Figure 6-7-c: The residual analysis for error ECC3 as a function of weight and torque with R-square 0.980

## Conclusion

In this study, a method was developed to investigate the effect of load on machine geometry with respect to the weight and torque resulting from the loading of a heavy workpiece. To this end, some heavy blocks (to achieve maximum weight) and some spacers (to achieve maximum torque) were designed and installed on the machine. G-code was generated for each experiment for the probing process. The raw data from the machine was processed using RUMBA program and the errors were analyzed. Using processed data, the polynomial graph for each error parameter was plotted for different loading conditions. Then, the correlation matrix between the error coefficients and weight and torque was defined. With this matrix, it was possible to see how much each error coefficient correlated with weight and torque and also with other error coefficients. Each error coefficient was modeled as a function of weight, torque and weight and torque. Finally, the residual analysis for each curve fitting was presented to verify the difference between the estimated value and the modeling value of the error coefficients to validate the model.

### Polynomial graphs

After processing the data of the machine, the polynomial graphs for each error parameter for different loading conditions were plotted for comparison. Most of the errors increased with increasing load. However, there were some errors which had a descending trend. For instance, EXX, EZX, EZZ and EZC errors decreased by adding more loads. There were also some other errors which were not affected significantly by increasing loads such as EYZ, EAZ, ECZ and EZB. The other errors like EAY, EBY, ECY, EBZ and the spindle errors were considered zero (they were negligible). One reason for decreasing the error by adding loads can be due to the fact that the weight vector and the initial unloaded error vector may be in opposite directions and neutralize these types of errors.

The maximum value of the error parameters was 0.5 mm which belonged to the error EZB, radial motion error of B axis in Z direction. The smallest absolute value belonged to the error EBC, tilt error motion of C axis around Y axis, which was equal to 0.05  $\mu\text{m}$ . Thus, the maximum error occurred between B and Z axis when the pallet rotates from  $B = -90^\circ$  to  $B = 90^\circ$ . The minimum

error occurred between C and Y axis when the pallet rotates from  $C = -180^\circ$  to  $C = 180^\circ$  and no significant error occurred in Y direction.

### **Correlation**

As it is discussed in chapter 6, the error coefficient EYY0, the zero degree of the positioning error of Y axis, had the highest dependency to the weight (with the correlation of 0.505) and the error coefficient EZZ2, the second degree of the positioning error of Z axis, had the highest dependency to the torque (with the correlation 0.685). The results show that weights and torques had more significant effects on the positioning error than on the other types of errors and these positioning errors were directly correlated with weight and torque. EZZ1 and EYX3 were also reversely correlated with weight and torque, respectively. Their values of dependency were equal to -0.573 and -0.746, respectively. In contrast, the third degree of the tilt error motion of C axis around Y axis, had the lowest dependency to the weight (with the correlation of 0.002) and the error coefficient EBX3, the third degree of the angular error of X axis around Y axis (pitch error), had the lowest dependency to the torque (with the correlation 0.030). This means that the third degree of the error coefficient had small dependency to weight and torque and these two parameters (W and T) did not have any significant effect on the cited errors.

The correlation factor between weight and torque was equal to 0.829 as expected, because these two parameters are linked together, torque is a function of weight and distance.

### **Curve Fitting**

All the error coefficients were modeled as a function of weight and torque to estimate the value of the error for different loading conditions without doing further tests. However, all the tests were done on the HU40T five-axis CNC machine tool. Thus, the model is only valid for this machine and its application for other machine tools even with the same topology has not been proven.

Since the real polynomial degree for each error coefficient was unknown, the degree of the best curve fitted with the highest R-square factor was chosen. The 3<sup>rd</sup> degree polynomial was used to model the errors. There were four coefficients for the functions with one variable (weight or torque) and 10 coefficients for the functions with two variables (both weight and torque).

The models revealed the following results:

1. Most of the zero degree errors (EYY0, EAB0, EBB0, ECX0, ECZ0, EXB0) had the same trend, ascending trajectory and descending trajectory by increasing torque and weight, respectively. However, two of them (EAX0 and EAZ0) had a direct relation with weight and a reverse relation with torque.
2. Most of the first degree of linear errors were decreased by torque and increased by weight. The first degree of linear positioning errors (EYY1, EZZ1) had the same trend. On the other side, most of the first degree angular errors were increased by torque and decreased by weight (EZB1, EYC1, EAB1, EAC1).
3. The second degree errors had different patterns for different loading conditions, although the second degrees of linear positioning errors (EYY2 and EZZ2) were similar and they decreased by torque and increased by weight.
4. The third degree of linear positioning errors (EXX3, EYY3, EZZ3) had the same trend, they were increased by torque and decreased by weight. The other 3rd degree errors had different trends.
5. The error coefficients EBB4 and ECC4 for the range of errors EBB (angular positioning error of B axis) and ECC (angular positioning error of C axis) respectively, had completely different trends. EBB4 was increased by torque and decreased by weight, whereas ECC4 was decreased by torque and increased by weight.

Overall, some error coefficients had a direct relation with torque and weight and some had an inverse relation. The value of the error coefficient was barely increased by both weight and torque. Although, these two factors (weight and torque) are linked together and have a direct relation, they had different effects on the error parameters.

### **Residual analysis**

The residual analysis shows the difference between the data set and the model. The smaller the difference is, the higher the accuracy of the curve fitting. Because there was no exact polynomial function for each error coefficient, the best fitted polynomial curve with the highest R-square value was chosen. In most cases, the value of the R-square was around 0.9 and the value of the residual was close to zero.

The number of the polynomial coefficients is increased by increasing the degree of the polynomial functions. In our modeling (with two variables, weight and torque, simultaneously), the 3<sup>rd</sup> degree polynomial with 10 coefficients was used. The values of these coefficients in the Modeling table (presented in appendix VI) were very small according to the small values of the error coefficients.

The modeling process was completed with correlation matrix, curve fitting and residual analysis. The model enables us to estimate the error coefficient values as a function of weight independently, torque independently and weight and torque simultaneously.

In conclusion, adding load on the machine increases the value of some error parameters and decreases the values of the other error parameters depending on the directions of the weight vector and initial unloaded error vector. In addition, the weight and the torque have different effect on the error coefficients. Although, weight and torque are directly proportional, some error coefficients are affected differently by weight and torque. It can be also concluded that the positioning errors are most affected by weight and torque values.

This knowledge can be further used in error compensation to increase the precision of the manufacturing. However, the results of this study could be more accurate if the other source of errors such as high temperature of the machine, vibration, internal forces, NC interpolation, etc is also considered. It is recommended to continue this study with more loading conditions to achieve more data points, simulate the cutting forces during probing and keep the environment in the same condition.

## References

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## Appendix

### Appendix I (Matlab programs)

#### 1. RUMBA PROGRAM [2]

```

clear all
%%Scale bar (2 balls) followed by the RUMBA.
workpiece_twists_nom_balls=[120  -120  326.2  0  0  0;...
                             -80  -160  177.8  0  0  0;...
                             -120  120  326.2  0  0  0;...
                             160   160  127   0  0  0];

Nballs=size(workpiece_twists_nom_balls,1); % includes RUMBA and scale bar
balls
ball_id=1:size(workpiece_twists_nom_balls,1); % defines which
workpiece_tool_twists are for ball artefacts.
ball_radii=ones(Nballs,1)*6.35; % Each ball has a radius to be defined. Here
they are all equal.
%ball_radii=[6.35; 6.35; 6.35; 6.35; 6.35; 6.35];% Each ball has a radius to
be defined. Here they are all potentially different.

%% Graph production requested
produce_graphs_flag=1; %1: Graph are produced; 0: No graph produced.

%% Twist of the faces to be probed even if only probed once. Defined as the
point to touch in the last workpiece joint frame and the alpha, beta, gamma
rotations use
% the order gamma then beta then alpha to align the z axis with the normal to
the face.
workpiece_twists_nom_faces=[]; % Last three components are in degrees.
Nfaces=size(workpiece_twists_nom_faces,1);

%% Scale bar.
Rumba_or_Samba=0; %State (0=Uncalibrated artefact; 24=Calibrated artefact made
of balls 2 and 4)
scale_bar_length=3.046686e+002; %Scale bar length, even if not used.

%% The indexations of the rotary axes to be used.
A=zeros(42,1);0;90;180;270];
Bvalues=[90 45 0 -45 -90 -60 -30 30 60 75 0 0 0 0 0 0];
B=[Bvalues(1);...
   Bvalues(2);Bvalues(2);...
   Bvalues(3);Bvalues(3);Bvalues(3);Bvalues(3);...
   Bvalues(4);Bvalues(4);...
   Bvalues(5);...
   Bvalues(6);Bvalues(6);...
   Bvalues(7);Bvalues(7);Bvalues(7);...
   Bvalues(8);Bvalues(8);Bvalues(8);...
   Bvalues(10);Bvalues(10);...
   Bvalues(11);Bvalues(11);Bvalues(11);Bvalues(11);...
   Bvalues(12);Bvalues(12);Bvalues(12);Bvalues(12);...
   Bvalues(13);Bvalues(13);Bvalues(13);Bvalues(13);...
   Bvalues(14);Bvalues(14);Bvalues(14);Bvalues(14);...
   Bvalues(15);Bvalues(15);Bvalues(15);Bvalues(15);...

```

```

    Bvalues(16);Bvalues(16);Bvalues(16);Bvalues(16)
];
Cvalues=[180 90 0 -90 -180 -135 -45 45 135 170 180 90 0 -90 -180 0 ];
C=[Cvalues(1);...
    Cvalues(2);Cvalues(2);...
    Cvalues(3);Cvalues(3);Cvalues(3);Cvalues(3);...
    Cvalues(4);Cvalues(4);...
    Cvalues(5);...
    Cvalues(6);Cvalues(6);...
    Cvalues(7);Cvalues(7);Cvalues(7);...
    Cvalues(8);Cvalues(8);Cvalues(8);...
    Cvalues(9);Cvalues(9);...
    Cvalues(10);Cvalues(10);...
    Cvalues(11);Cvalues(11);Cvalues(11);Cvalues(11);...
    Cvalues(12);Cvalues(12);Cvalues(12);Cvalues(12);...
    Cvalues(13);Cvalues(13);Cvalues(13);Cvalues(13);...
    Cvalues(14);Cvalues(14);Cvalues(14);Cvalues(14);...
    Cvalues(15);Cvalues(15);Cvalues(15);Cvalues(15);...
    Cvalues(16);Cvalues(16);Cvalues(16);Cvalues(16)
];

abc_values=[A B C];

artefact_id=[4;...
    3;4;...
    1;2;3;4;...
    3;4;...
    2;...
    3;4;...
    1;3;4;...
    2;3;4;...
    1;4;...
    1;4;...
    1;2;3;4;...
    1;2;3;4;...
    1;2;3;4;...
    1;2;3;4;...
    1;2;3;4;...
    1;1;1;1
];% The identification number of the balls and faces to be probed at each
indexation set.

%% Prepare the indexation for each artefact probing operation and also the
sequence of artefact identifier for the whole process.

%% Tool definition for each artefact probing operation
tool_id=ones(size(artefact_id,1),1); % Each time a ball or face is measured
this contains the tool identifier.
tool_length(1)=100;

%% Process conditions for face probing
approach_distance=10; % Expected travelled distance to touch.
overtravel_distance_to_touch=5; % By how much the machine carries on searching
for a touch beyond the expected touch.
retract_distance=1; % The probe comes back after the touch by this distance.

```

```

stylus_tip_radius=3;

%% Define by a twist of three translations in x,y and z and three rotations
around x, y
% and z. The first twist is from the foundation frame to the first joint.
% The tool and workpiece are not considered.
%
tool_branch_link_twists_error_actual=tool_branch_link_twists_error_nom+0.0000*
(randn(ntb,6)-.5);
%
workpiece_branch_link_twists_error_actual=workpiece_branch_link_twists_error_n
om+0.0000*(randn(nwb,6)-.5);
tool_branch_link_twists_error_actual=...
[      0      0      0;...
      0      0      0;...
      0      0      0;...
      0      0      0;...
      0      0      0;...
      0      0      0]'; % Twists are columnwise
before transpose action.
workpiece_branch_link_twists_error_actual=...
[      0      0 0;...
      0      0 0;...
      0      0 0;...
      0      0 0;...
      0      0 0;...
      0      0 0]'; % Twists are columnwise before transpose
action.

% Motion errors coefficients for the polynomials (starting with
% the zero degree)and the backlash coefficient (an offset) for the six
components
% of error for the six axes X, Y, Z, A, B, C (even those not present on the
% machine.
% Each column is for one polynomial, i.e one motion error component
% of one axis. Points are generated and a polynomial fitted.
% [EXX0 EXX1 EXX2 EXX3 EXX4 EXXR ;EYX0 EYX1 EYX2 EYX3 EYX4 EYXR ; EZX0
EZX1 EZX2 EZX3 EZX4 EZXR ; EAX0 EAX1 EAX2 EAX3 EAX4 EAXR ; EBX0 EBX1
EBX2 EBX3 EBX4 EBXR; ECX0 ECX1 ECX2 ECX3 ECX4 ECXR;
% [EXY0 EXY1 EXY2 EXY3 EXY4 EXYR ;EYY0 EYY1 EYY2 EYY3 EYY4 EYYR ; EYZ0
EYZ1 EYZ2 EYZ3 EYZ4 EYZR ; EAY0 EAY1 EAY2 EAY3 EAY4 EAYR ; EBY0 EBY1
EBY2 EBY3 EBY4 EBYR; ECY0 ECY1 ECY2 ECY3 ECY4 ECYR;
% [EXZ0 EXZ1 EXZ2 EXZ3 EXZ4 EXZR ;EYZ0 EYZ1 EYZ2 EYZ3 EYZ4 EYZR ; EZZ0
EZZ1 EZZ2 EZZ3 EZZ4 EZZR ; EAZ0 EAZ1 EAZ2 EAZ3 EAZ4 EAZR ; EBZ0 EBZ1
EBZ2 EBZ3 EBZ4 EBZR; ECZ0 ECZ1 ECZ2 ECZ3 ECZ4 ECZR;
% [EXA0 EXA1 EXA2 EXA3 EXA4 EXAR ;EYA0 EYA1 EYA2 EYA3 EYA4 EYAR ; EZA0
EZA1 EZA2 EZA3 EZA4 EZAR ; EAA0 EAA1 EAA2 EAA3 EAA4 EAAR ; EBA0 EBA1
EBA2 EBA3 EBA4 EBAR; ECA0 ECA1 ECA2 ECA3 ECA4 ECAR;
% [EXB0 EXB1 EXB2 EXB3 EXB4 EXBR ;EYB0 EYB1 EYB2 EYB3 EYB4 EYBR ; EZB0
EZB1 EZB2 EZB3 EZB4 EZBR ; EAB0 EAB1 EAB2 EAB3 EAB4 EABR ; EBB0 EBB1
EBB2 EBB3 EBB4 EBBR; ECB0 ECB1 ECB2 ECB3 ECB4 ECBR;
% [EXC0 EXC1 EXC2 EXC3 EXC4 EXCR ;EYC0 EYC1 EYC2 EYC3 EYC4 EYCR ; EZC0
EYC1 EYC2 EYC3 EYC4 EYCR ; EAC0 EAC1 EAC2 EAC3 EAC4 EACR ; EBC0 EBC1
EBC2 EBC3 EBC4 EBCR; ECC0 ECC1 ECC2 ECC3 ECC4 ECCR]'
motion_error_coef_actual=...
[0 0.0000112 0.000000113 0.000000000114 0 ; 0 0 0.000000123 -0.00000000124
0 ; 0 0 -0.000000133 0.00000000134 0 ; ...

```

```

0.000141 -0.00000000142 +0.0000000000143 -0.00000000000000144 0 ; 0
+0.00000000152 -0.0000000000153 +0.00000000000000154 0; 0.000161 -
0.0000000000162 +0.000000000000163 -0.00000000000000164 0;...

...
0.0211 0 -0.00000213 0.00000000214 0 ; 0.0221 -0.000222
0.00000223 0.00000000224 0 ; 0 0 0.00000233 -0.00000000234 0 ;...
0 -0.00000000242*0 +0.000000000243*0 -0.00000000000000244*0 0
; 0 0*0.00252 0*0.00000000253 0*0.0000000000254 0; 0 0 0 0 0;...

...
0 -0.00000312 0.000000313 -0.00000000314 0 ; 0 0 0.00000000323
0.00000000324 0 ; 0 0.0000332 0.000000333 -0.00000000334 0 ; ...
0.000341 +0.00000000342 -0.0000000000343 +0.00000000000000344 0 ; 0
0*0.00352 0*0.000000353 0*0.0000000000354 0; 0.000361 0.0000000000362 -
0.000000000000363 0.000000000000364 0;...

...
0 0 0 0 0 ; 0 0 0 0 0 ; 0 0 0 0 0 ; ...
0 0 0 0 0 ; 0 0 0 0 0; 0 0 0 0 0;...

...
0.00511 +0.00000512 -0.00000513 +0.0000000514 0 ; 0
0.00000522 0.00000523 0.0000000524 0 ; 0 0.0000532 0.00000533 0.0000000534
0 ;...
0.000541 -0.0000000542 +0.00000000543 -0.0000000000544 0 ; 0.000551
0.00000000552 0.00000000553 0.0000000000554 0.000555; 0 0.000000562
0.000000563 0.0000000564 0;...

...
0 -0.000612 0.000000613 -0.000000614 0 ; 0 0.000622 -
0.0000000623 0.0000000624 0 ; 0 -0.00632 0.000000633 0.0000000634 0
;...
0 -0.000642 0.0000643 -0.00000644 0 ; 0 0.000652 -0.0000000653 -
0.000000000654 0; 0 0.000662 0.0000663 0.00000664 0.000665]'

```

```

mt_para_motion_def = [1 4 1; 1 6 1; 5 1 1; 5 4 1; 5 5 1; 3 1 2; 3 4 1; 3 6
1;... % The classical 8 parameters.

```

```

% mt_para_motion_def = [ 2 5 1; 2 4 1; 1 3 2; 4 5 1; 4 4 1; 4 2 1; %
The classical 8 parameters.

```

```

2 1 1 ;2 2 1;... % Spindle location.
2 2 2; 3 3 2;... % Scale Y and Z relative to X.
%% 1 1 2 % X scale is automatically included (cond=)

```

```

1 1 3;
1 1 4;... % X-axis scale 2nd and 3rd degree terms.
2 2 3;
2 2 4;... % Y-axis scale 2nd and 3rd degree terms.
3 3 3;
3 3 4;... % Z-axis scale 2nd and 3rd degree terms.
5 5 2; % B-axis B angular positioning, 1st,
5 5 3;
5 5 4;... % B-axis B angular positioning, 2nd and 3rd degree.
6 6 2 % C-axis angular positioning,1st degree term,
6 6 3;
6 6 4;% C-axis angular positioning, 2nd and 3rd degree.
1 2 3;
1 2 4;... % X-axis straightness in y, 2nd and 3rd degree terms.
1 3 3;
1 3 4;... % X-axis straightness in z, 2nd and 3rd degree terms.

```

```

%% 2 1 2; % Couple with the Y to Z out-of-squareness.
    2 1 3;
    2 1 4;... % Y-axis straightness in x, 2nd and 3rd degree terms.
    2 3 3;
    2 3 4;... % Y-axis straightness in z, 2nd and 3rd degree terms.
    3 1 3;
    3 1 4;... % Z-axis straightness in x, 2nd and 3rd degree terms.
    3 2 3;
    3 2 4;... % Z-axis straightness in y, 2nd and 3rd degree terms.
    1 4 2;
    1 4 3;
    1 4 4;... % X-axis angular error around x, 1st, 2nd and 3rd degree terms.
    1 5 2;
    1 5 3;
    1 5 4;... % X-axis angular error around y, 1st, 2nd and 3rd degree terms.
    1 6 2;
    1 6 3;
    1 6 4;... % X-axis angular error around z, 1st, 2nd and 3rd degree terms.
%%    2 4 2; 2 4 3; 2 4 4; % Y-axis angular error around x, 1st, 2nd and 3rd
degree terms. Two tools are needed to distinguish from straightness orscale
termes
%%    2 5 2; 2 5 3; 2 5 4; % Y-axis angular error around y, 1st, 2nd and 3rd
degree terms.Two tools are needed to distinguish from straightness orscale
termes
%%    2 6 2; 2 6 3; 2 6 4; % Y-axis angular error around z, 2nd and 3rd degree
terms.Two tools are needed to distinguish from straightness orscale termes
    3 4 2;
    3 4 3;
    3 4 4; % Z-axis angular error around x, 1st, 2nd and 3rd degree terms.
(cond=113.2082)
%%    3 5 2; 3 5 3; 3 5 4; % Z-axis angular error around y, 1st, 2nd and 3rd
degree terms. (two tool length are needed). Not strong because the bryan
length is constant to produce a straightness
    3 6 2;
    3 6 3;
    3 6 4;... % Z-axis angular error around z, 1st, 2nd and 3rd degree terms.
(cond=113.2979)
    5 1 2;
    5 1 3;
    5 1 4;... % B-axis radial (x) motion, 2nd and 3rd degree. (cond=113.5946)
    5 2 2;
    5 2 3;
    5 2 4; % B-axis axial (y) motion, 2nd and 3rd degree.
(cond=116.9237)
    5 3 2; %(cond=116.9346)
    5 3 3; %(cond=277.2891)
    5 3 4; % B-axis radial (z) motion, 2nd and 3rd degree.
(cond=327.4958)
    5 4 2; % (cond=304.6686)
    5 4 3; % (cond=334.3825)
    5 4 4;... % B-axis A tilt, 1st, 2nd and 3rd degree. (cond=335.0143)

    5 6 2; % (cond=335.4621)
    5 6 3; % (cond=335.9265)
    5 6 4;... % B-axis C-tilt, 1st, 2nd and 3rd degree. (cond=10E+15).
Probably couples with 6 6 4 but why not 562 and 563.

```

```

6 1 2; % (cond=352.5610)
6 1 3; % (cond=520.7955)
6 1 4;... % C-axis radial (x) motion, 1st, 2nd and 3rd degree.
(cond=551.7340)
6 2 2; % (cond=1011.9)
6 2 3; % (cond=1015.5)
6 2 4; % C-axis radial (y) motion, 1st, 2nd and 3rd degree.
(cond=1039.5)
%
6 3 2; % (cond=1041.0)
%
6 3 3; % (cond=1051.9)
6 3 4; % C-axis axial motion, 2nd and 3rd degree. (cond=10+15)
6 4 2; % (cond=1052.3)
6 4 3; % (cond=1056.2)
6 4 4; % C-axis yaw, 1st, 2nd and 3rd degree.
%
6 5 2; % (cond=1064.3)
6 5 3; % (cond=1066.1)
6 5 4; % C-axis yaw, 1st, 2nd and 3rd degree. (cond=1085.9)
%
1 1 5;
%
2 2 5;...% Backlash on the EXX
%
3 3 5;... %Z-axis backlash.
%
4 4 5; % A-axis indexation backlash.
5 5 5; % B-axis indexation backlash. (cond=1561.1)
6 6 5% C-axis indexation backlash. (cond=high and variable, no convergence
to a solution)
];

wb_link_para_def=[]; % All workpiece branch link errors can be selected
(index 1 is the axis order in the branch and index 2 is the error (xyzabc))
tb_link_para_def=[]; % All tool branch link errors can be selected

[motion_error_coef_estimation_error,
tool_branch_link_twists_error_estimation_error,
workpiece_branch_link_twists_error_estimation_error,...
tool_twists_estimation_error, workpiece_twists_estimation_error,
mean_vol_err_norm_wtot_fullmodel, max_vol_err_norm_wtot_fullmodel] = ...
SAMBA_sim_v9_8_2_4_7_1_HU40(tool_branch_link_twists_error_actual,
workpiece_branch_link_twists_error_actual,...
motion_error_coef_actual, workpiece_twists_nom_balls,
workpiece_twists_nom_faces, mt_para_motion_def, wb_link_para_def,
tb_link_para_def,...
abc_values, artefact_id, tool_length, tool_id, stylus_tip_radius,
approach_distance, overtravel_distance_to_touch, retract_distance,
scale_bar_length,...
Rumba_or_Samba, ball_radii, produce_graphs_flag);

load('junk');
mean_vol_err_norm_wtot_fullmodel
max_vol_err_norm_wtot_fullmodel
cond_J_reduced_normalised=cond(J_reduced_normalised)
rankJ_reduced_normalised=rank(J_reduced_normalised)
Nbr_coef_estimated=size(mt_para_motion_def,1)+1
max(max(abs(motion_error_coef_estimation_error)))

```

## 2.POLYNOMIAL GRAPHS FOR EACH RANGE OF ERROR

```
clear all
clc
```

```
error={'EXX'          'EBY'          'EZA'          'EXC'
      'EYX'          'ECY'          'EAA'          'EYC'
      'EZX'          'EXZ'          'EBA'          'EZX'
      'EAX'          'EYZ'          'ECA'          'EAC'
      'EBX'          'EZZ'          'EXB'          'EBC'
      'ECX'          'EAZ'          'EYB'          'ECC'};
      'EXY'          'EBZ'          'EZB'
      'EYY'          'ECZ'          'EAB'
      'EZY'          'EXA'          'EBB'
      'EAY'          'EYA'          'ECB'

test= [0      0      0      -0.000474681      0      -0.000421654      -0.189273502
0.043597012 0      0      0      0      0      0      0.000204604 0      0.000580163 0      0      0
0      0      0      -0.19776664 0      0      0.000437287 6.40E-06      0      0      0      0      0
0;
0      0      0      1.80E-09      4.03E-09      -2.82E-08      0      2.64E-05      0      0      0      0
-0.000191839      0      7.35E-05      -3.62E-06      0      -1.03E-05      0      0      0      0      0
0      -0.199878102      -0.014256293      -0.095944374      -0.000466226      -4.41E-05
0.000439031 -0.00330205 0.002404289 8.95E-05      3.19E-06      1.08E-05      -
4.86E-06;
8.11E-08      3.02E-07      -1.76E-07      1.14E-09      -1.28E-09      1.07E-09      4.67E-
08      -2.55E-07      -1.64E-07      0      0      0      8.59E-07      -1.02E-07      -2.15E-07
1.53E-08      0      5.85E-08      0      0      0      0      0      0.03573853 -0.000138738
-0.091190372 -0.000165583 1.37E-05      -0.000166829      0.000216177
0.000855978 -7.36E-06      1.72E-06      -1.94E-07      -4.31E-06;
6.97E-11      1.46E-09      1.31E-09      -3.15E-13      2.66E-12      1.09E-13      -
2.70E-09      6.51E-10      4.64E-10      0      0      0      -1.01E-09      7.92E-11      2.17E-
11      -2.03E-11      0      -1.07E-10      0      0      0      0      0      0.018371271
0.011023751 0.013851252 9.44E-05      4.71E-05      -8.80E-05      0.000340329 -
0.000243436      6.30E-06      -3.72E-07      -1.11E-06      5.07E-07;
0      0      0      0      0      0      0      0      0      0      0      0      0      0
0      0      0      0      0      0      0      2.16E-06      0      0      0      0      0      -2.91E-05;

0      0      0      -0.000401835      0      -0.000232339      -0.177515773      0.029239426 0
0      0      0      0      0      0      2.23E-05      0      2.85E-05      0      0      0      0      0      -
0.177223121      0      0      0.000333524 -5.89E-06      0      0      0      0      0      0      0;
0      0      0      1.99E-07      -2.76E-07      -1.14E-07      0      -5.54E-05      0      0      0      0
-3.67E-05      0      -7.39E-05      -6.37E-08      0      -7.24E-08      0      0      0      0      0
-0.198957467 -0.002346157 -0.076991594 -0.000225889 -2.62E-05
0.0003998      0.000879362 0.000566728 -0.000372887      2.43E-07      -8.74E-06      -
3.93E-06;
-6.51E-08      2.12E-07      7.19E-07      2.11E-09      -1.34E-09      1.48E-09      -
1.82E-08      -1.16E-07      -3.67E-07      0      0      0      3.66E-08      7.85E-07      -2.76E-
07      2.42E-09      0      7.66E-10      0      0      0      0      0      0.02425184
0.001935952 -0.08179698 -0.000126204      4.73E-05      -0.000107405
0.000239676 -0.000938318      0.000173012 -4.99E-06      -4.06E-07      -6.57E-06;
-2.01E-09      1.69E-09      1.93E-09      -7.10E-12      -1.13E-12      4.57E-12      -
1.26E-09      3.71E-10      -1.02E-09      0      0      0      3.10E-10      -3.97E-09      -2.11E-
10      -2.05E-11      0      -8.61E-12      0      0      0      0      0      0.02604611 -
0.001164638      0.009203618 1.57E-05      6.08E-06      -5.86E-05      -6.54E-05      -
6.98E-05      3.71E-05      -7.10E-08      8.36E-07      -4.40E-07;
```

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 -1.30E-06 0 0 0 0 0 0 -1.25E-05;

0 0 0 -0.000372637 0 -9.10E-05 -0.263547837 0.00398384 0 0
0 0 0 0 0 3.28E-05 0 -1.15E-05 0 0 0 0 0 0 -
0.249174175 0 0 0.000336047 1.13E-05 0 0 0 0 0 0 0;
0 0 0 1.39E-08 -2.02E-07 -8.72E-08 0 0.000593751 0 0 0 0
-4.90E-05 0 0.000487567 1.75E-07 0 1.32E-07 0 0 0 0 0 0
-0.091657582 -0.002604597 -0.145774797 -8.90E-05 -8.87E-06
0.000347999 -0.000655504 0.002405319 0.000242007 3.54E-06 1.29E-06
9.25E-07;
6.12E-09 2.01E-07 2.79E-07 1.04E-09 -6.50E-10 -3.43E-10 -
5.01E-08 1.24E-08 -2.30E-07 0 0 0 3.44E-07 9.77E-08 1.22E-
06 -2.06E-09 0 -6.49E-10 0 0 0 0 0 0 0.055123214 -
0.001082645 -0.033965073 -0.000136657 -1.61E-05 -2.69E-05 -5.00E-
05 -0.000248414 1.49E-05 -1.10E-06 1.83E-06 -4.77E-06;
2.81E-09 -1.92E-10 8.25E-10 -1.18E-12 2.13E-12 1.94E-12 -
1.08E-09 -4.31E-09 -1.10E-09 0 0 0 -9.59E-10 -5.79E-10 -6.32E-
09 2.25E-13 0 2.57E-12 0 0 0 0 0 0 0.011965572
0.001406533 0.019255922 7.33E-06 3.16E-06 -4.72E-05 7.18E-05 -
0.000230745 2.30E-05 -4.12E-07 -1.50E-07 -4.80E-07;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 2.30E-06 0 0 0 0 0 0 -3.14E-05;

0 0 0 -2.73E-05 0 -0.000479736 -0.19126993 0.002170646 0 0 0
0 0 0 0 6.64E-05 0 -7.15E-06 0 0 0 0 0 0 -
0.180546421 0 0 8.72E-05 3.52E-06 0 0 0 0 0 0 0;
0 0 0 1.44E-08 -7.58E-08 -6.68E-09 0 0.000609577 0 0 0 0
-0.000100714 0 0.000487778 6.83E-07 0 -9.12E-07 0 0 0 0 0
0 0.012720365 -0.004432425 -0.075866762 -0.000480463 2.43E-05
0.00011788 0.004125531 0.001035647 0.000182866 5.57E-08 -1.59E-05 -
4.40E-07;
4.89E-08 -6.24E-10 -2.40E-08 2.21E-11 -6.17E-10 -5.56E-10 1.11E-
08 6.82E-08 -1.16E-07 0 0 0 6.00E-07 -4.40E-07 1.11E-06
-6.43E-09 0 8.79E-09 0 0 0 0 0 0 0.026661557 -0.003912391
0.017601666 -3.86E-05 -3.05E-05 -0.000189853 9.03E-05 -0.000258836
-0.000281166 -1.08E-06 4.61E-07 -5.82E-06;
2.70E-09 -5.76E-10 6.09E-10 -1.36E-12 2.36E-12 1.25E-13 -
1.44E-09 -4.47E-09 -2.10E-10 0 0 0 -1.07E-09 7.56E-10 -5.43E-
09 1.47E-11 0 -1.99E-11 0 0 0 0 0 0 0.006888218 -
0.0003462 0.010192869 5.90E-05 -3.70E-05 -3.85E-05 -0.00040978 -
0.000103826 -2.39E-05 1.63E-08 1.62E-06 -2.82E-07;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1.90E-06 0 0 0 0 0 0 -3.32E-05;

0 0 0 -0.000209353 0 -0.00087738 -0.1882583 0.019832062 0 0 0
0 0 0 0 3.21E-05 0 1.33E-06 0 0 0 0 0 0 -
0.179919138 0 0 0.000288941 -1.69E-05 0 0 0 0 0 0 0;
0 0 0 1.71E-08 6.26E-08 3.98E-08 0 3.67E-05 0 0 0 0
6.76E-05 0 8.50E-05 -4.14E-07 0 4.48E-07 0 0 0 0 0 0
-0.184244144 -0.01006325 -0.082270814 -0.000946659 -1.90E-05
0.000317449 -0.001979041 0.007694845 -0.000619898 3.75E-05 -4.91E-06
-3.65E-06;
-1.40E-07 1.79E-08 -1.20E-07 -1.42E-09 5.12E-10 -5.71E-10 8.47E-
08 -1.26E-07 -4.17E-08 0 0 0 -7.21E-07 -7.13E-07 -1.48E-08
5.67E-09 0 -1.84E-08 0 0 0 0 0 0 0.02821871 -0.00048406 -

```



```

0.079472912      -0.000127532      2.24E-05      -0.000353873      0.000800595
0.000223002 -6.17E-05      -2.92E-06      -2.32E-06      -5.58E-06;
6.06E-10      8.39E-10      -6.20E-10      -1.60E-12      -1.25E-13      -4.77E-13      -
5.07E-09      8.01E-10      9.42E-10      0      0      0      2.60E-09      1.46E-09      -6.89E-
10      -1.13E-11      0      8.69E-11      0      0      0      0      0      0.022493314
0.006495823 0.013353582 0.000155997 1.60E-05      -5.16E-05      0.000200468 -
0.000811836      4.93E-05      -3.99E-06      5.15E-07      -4.39E-08;
0      0      0      0      0      0      0      0      0      0      0      0      0      0      0      0
0      0      0      0      0      0      0      0      7.09E-06      0      0      0      0      0      0      -2.68E-05;

0      0      0      -0.000369776      0      -0.000817683      -0.177712038      0.028583184 0
0      0      0      0      0      0      -1.38E-06      0      6.29E-05      0      0      0      0      0      0      0      -
0.171594509      0      0      0.000273758 -1.11E-05      0      0      0      0      0      0      0      0;
0      0      0      5.26E-09      -1.46E-07      -1.06E-07      0      -3.01E-05      0      0      0      0      0
-3.13E-05      0      -6.10E-05      -2.42E-07      0      -7.61E-07      0      0      0      0      0      0
-0.072236234      -0.001920204      -0.072707098      -0.000799987      2.19E-05
0.000314724 0.001243344 0.00311529 -0.000705044      6.60E-06      -1.39E-05
1.52E-06;
-1.62E-08      1.90E-07      1.04E-06      1.89E-09      -7.16E-10      1.39E-09      -
1.98E-08      -6.35E-08      -7.56E-07      0      0      0      2.44E-07      9.06E-07      -2.88E-
07      3.78E-09      0      7.03E-09      0      0      0      0      0      0.022872035
0.002281952 -0.022807065      -0.00010072 4.90E-05      -0.000362519
0.000703277 -0.000860503      0.000163463 -4.50E-06      -2.48E-06      -4.37E-06;
-1.45E-09      1.53E-09      1.17E-09      1.43E-12      -1.78E-12      3.64E-12      -
1.21E-09      4.60E-11      -1.22E-09      0      0      0      -6.25E-10      -3.29E-09      -1.24E-
09      -1.95E-11      0      -2.37E-11      0      0      0      0      0      0.013992917 -
0.001962294      0.009057396 9.37E-05      -3.57E-05      -4.08E-05      -0.000114312
-0.000330226      7.26E-05      -7.05E-07      1.46E-06      -5.66E-07;
0      0      0      0      0      0      0      0      0      0      0      0      0      0      0      0
0      0      0      0      0      0      0      0      -1.62E-06      0      0      0      0      0      0      -2.39E-05;

0      0      0      -0.000480661      0      -0.000212592      -0.19687035 0.036362973 0      0
0      0      0      0      0      8.40E-06      0      -6.66E-05      0      0      0      0      0      0      0      -
0.198413302      0      0      0.000402499 -8.72E-06      0      0      0      0      0      0      0      0;
0      0      0      -2.93E-08      6.67E-08      -6.94E-08      0      3.20E-05      0      0      0      0      0
-2.58E-05      0      3.23E-05      1.03E-07      0      3.75E-07      0      0      0      0      0      0
-0.110025595      -0.006692352      -0.096055982      -0.00020758 -3.42E-06
0.000402143 0.00013579 0.003202358 0.001247311 2.40E-05      5.72E-07      -
2.33E-06;
3.05E-08      3.57E-07      2.50E-07      1.54E-09      -3.53E-10      -4.64E-10      2.58E-
08      -1.33E-07      -1.77E-07      0      0      0      -6.90E-08      4.12E-07      -3.60E-07
3.02E-10      0      -1.20E-10      0      0      0      0      0      0      0.035267759 -7.85E-05      -
0.045786892      -0.000154452      1.64E-05      -7.42E-05      -0.000864658      -
0.00063209 0.00031627 -2.90E-06      2.98E-06      -3.88E-06;
1.06E-11      -6.91E-10      6.17E-10      1.94E-12      -1.22E-12      1.56E-12      -
1.64E-09      2.00E-10      -1.49E-09      0      0      0      4.34E-10      -1.26E-09      6.07E-
10      -8.16E-12      0      -2.54E-12      0      0      0      0      0      0.015211084
0.003099853 0.010752721 3.48E-05      1.38E-06      -5.77E-05      -3.23E-05      -
0.0003294 -0.000116159      -2.44E-06      7.59E-08      -1.95E-07;
0      0      0      0      0      0      0      0      0      0      0      0      0      0      0      0
0      0      0      0      0      0      0      0      4.69E-06      0      0      0      0      0      0      -2.68E-05;

0      0      0      -0.000471095      0      0.001938927 -0.282110448      0.024381622 0      0
0      0      0      0      0      -0.000664829      0      -0.000104903      0      0      0      0      0      0      0
-0.308136093      0      0      0.00022737 -2.05E-05      0      0      0      0      0      0      0      0;

```

```

0 0 0 5.06E-08 3.06E-07 -5.86E-08 0 0.000752264 0 0 0 0
-0.000380741 0 7.10E-05 8.79E-06 0 1.53E-06 0 0 0 0 0
0 -0.179917865 -0.004852355 -0.217304028 0.001953365 -1.54E-05
0.00021449 0.003127625 0.014145963 0.003085596 5.48E-05 1.08E-06 2.58E-
06;
4.67E-07 -1.08E-07 -3.13E-07 8.15E-10 -1.49E-09 -2.04E-09 -
7.44E-09 1.02E-07 -1.27E-08 0 0 0 9.00E-07 1.29E-06 4.23E-
06 -4.68E-08 0 -9.80E-09 0 0 0 0 0 0.095542491
0.000155785 -0.075329938 -5.81E-05 -7.92E-05 0.000891182 -0.001526723
-0.001256809 -0.000117711 -2.66E-06 4.98E-06 -6.28E-06;
5.96E-09 -2.20E-09 1.90E-09 -3.70E-12 -1.88E-12 -4.70E-12 -
7.11E-10 -4.42E-09 -1.12E-09 0 0 0 -4.23E-10 -2.12E-09 -1.04E-
08 7.28E-11 0 1.90E-11 0 0 0 0 0 0.019085576
0.003718637 0.040029129 -0.000314963 6.15E-06 -6.49E-05 -0.000328798
-0.001401716 -0.000220338 -5.55E-06 -1.00E-07 -1.06E-06;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1.27E-05 0 0 0 0 0 0 -1.71E-05;

0 0 0 -0.000145846 0 -0.000193697 -0.228939526 0.022221588 0
0 0 0 0 0 0 7.94E-05 0 5.07E-05 0 0 0 0 0 0 -
0.21837229 0 0 0.000224701 2.46E-06 0 0 0 0 0 0 0;
0 0 0 -7.08E-09 -1.03E-07 3.69E-08 0 2.12E-05 0 0 0 0
3.84E-05 0 -2.43E-05 -1.74E-06 0 -1.14E-06 0 0 0 0 0 0
-0.194302536 -0.008153058 -0.120767023 -0.000238487 -6.86E-05
0.000219587 -0.001212603 0.005703511 -0.000162094 1.70E-05 5.95E-06
1.23E-06;
-9.51E-08 -1.17E-07 -8.61E-08 -5.35E-10 -4.86E-10 1.11E-10 2.54E-
08 -6.15E-08 -1.31E-07 0 0 0 -1.41E-07 -5.36E-07 3.80E-07
1.10E-08 0 4.03E-09 0 0 0 0 0 0 0.042620785 -0.000810718
-0.088633537 -8.15E-05 2.73E-05 -7.17E-05 0.000540534 0.000174439 -
3.56E-05 8.56E-08 -3.38E-07 -5.19E-06;
2.54E-10 6.36E-10 1.64E-10 -1.86E-12 2.49E-12 -2.09E-12 -
3.37E-09 5.37E-10 1.37E-09 0 0 0 5.51E-10 1.15E-09 -1.30E-
09 -1.70E-11 0 4.50E-13 0 0 0 0 0 0.018262428 0.0047489
0.016799487 4.53E-05 4.78E-05 -3.66E-05 0.000129803 -0.000591548
2.48E-05 -1.85E-06 -6.52E-07 -4.77E-07;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 5.73E-06 0 0 0 0 0 0 -2.41E-05;

0 0 0 0.00039273 0 -0.000630268 -0.207620003 0.029508341 0 0
0 0 0 0 0 6.68E-05 0 0.000164865 0 0 0 0 0 0 -
0.259069545 0 0 -0.000342975 -0.000160107 0 0 0 0 0 0
0;
0 0 0 2.00E-08 2.36E-07 2.53E-08 0 4.82E-05 0 0 0 0
0.00011317 0 -2.27E-05 -3.08E-08 0 -3.05E-06 0 0 0 0 0 0
-0.106376184 0.00549894 -0.150859767 -0.000608053 -5.57E-06 -
0.000321411 0.0035022 -0.00042137 -0.000820169 -2.62E-05 -2.19E-05
-1.64E-06;
1.43E-08 6.20E-08 -1.06E-06 -3.45E-10 3.24E-09 2.80E-10 8.56E-
08 -1.01E-07 2.02E-07 0 0 0 -3.63E-07 -5.89E-07 4.89E-07
-5.18E-09 0 2.28E-08 0 0 0 0 0.054652515 -0.004620531
-0.031298608 0.000148041 -7.46E-06 -0.000299857 0.000900738 -
0.000236971 -0.000171749 -2.03E-06 -2.69E-07 -2.39E-06;
-1.39E-10 3.10E-10 -2.39E-09 -1.75E-12 5.61E-12 1.91E-12 8.66E-
10 -2.31E-10 -3.77E-10 0 0 0 7.79E-10 1.52E-09 -7.25E-10
1.84E-11 0 -4.73E-11 0 0 0 0 0 0 0.027898474 -0.011295043

```

```

0.016356029 5.20E-05 -6.94E-05 4.22E-05 -0.00036949 3.11E-05 8.02E-
05 2.64E-06 2.22E-06 5.06E-08;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 3.04E-06 0 0 0 0 0 0 0 -2.90E-05;

0 0 0 -0.000474681 0 -0.000421654 -0.189273502 0.043597012 0
0 0 0 0 0 0 0.000204604 0 0.000580163 0 0 0 0 0 0 0 -
0.19776664 0 0 0.000437287 6.40E-06 0 0 0 0 0 0 0 0;
0 0 0 1.80E-09 4.03E-09 -2.82E-08 0 2.64E-05 0 0 0 0 0
-0.000191839 0 7.35E-05 -3.62E-06 0 -1.03E-05 0 0 0 0 0
0 -0.199878102 -0.014256293 -0.095944374 -0.000466226 -4.41E-05
0.000439031 -0.00330205 0.002404289 8.95E-05 3.19E-06 1.08E-05 -
4.86E-06;
8.11E-08 3.02E-07 -1.76E-07 1.14E-09 -1.28E-09 1.07E-09 4.67E-
08 -2.55E-07 -1.64E-07 0 0 0 8.59E-07 -1.02E-07 -2.15E-07
1.53E-08 0 5.85E-08 0 0 0 0 0 0 0 0.03573853 -0.000138738
-0.091190372 -0.000165583 1.37E-05 -0.000166829 0.000216177
0.000855978 -7.36E-06 1.72E-06 -1.94E-07 -4.31E-06;
6.97E-11 1.46E-09 1.31E-09 -3.15E-13 2.66E-12 1.09E-13 -
2.70E-09 6.51E-10 4.64E-10 0 0 0 -1.01E-09 7.92E-11 2.17E-
11 -2.03E-11 0 -1.07E-10 0 0 0 0 0 0 0.018371271
0.011023751 0.013851252 9.44E-05 4.71E-05 -8.80E-05 0.000340329 -
0.000243436 6.30E-06 -3.72E-07 -1.11E-06 5.07E-07;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 2.16E-06 0 0 0 0 0 0 0 -2.91E-05;

0 0 0 -0.001082026 0 0.000463604 -0.295062368 0.060244988 0 0
0 0 0 0 0 -0.000358512 0 -0.00052546 0 0 0 0 0 0 0 -
0.278831594 0 0 0.000806837 -1.89E-05 0 0 0 0 0 0 0;
0 0 0 -2.30E-08 1.16E-07 -2.94E-07 0 9.46E-05 0 0 0 0 0
0.000144412 0 -0.00061426 3.16E-06 0 5.90E-06 0 0 0 0 0 0
0.184076322 -0.017680954 -0.182724515 0.000446075 6.15E-05
0.000736021 0.007532286 0.013704439 0.000183663 5.55E-05 -1.07E-05 3.14E-
06;
-2.17E-08 -5.70E-08 8.06E-08 5.51E-10 -1.72E-11 -2.15E-09 7.08E-
09 -1.66E-07 -1.57E-07 0 0 0 -7.62E-07 1.51E-06 3.22E-06
-1.78E-08 0 -2.64E-08 0 0 0 0 0 0 0.06259115 -0.00295784
0.097457623 -0.000362925 4.56E-05 0.000230106 -0.000741627 -
0.001430585 5.54E-05 -2.01E-06 2.43E-06 -5.93E-06;
1.87E-09 -3.38E-09 -5.91E-11 4.82E-13 -1.29E-12 4.87E-12 -
2.76E-09 -5.79E-11 -2.05E-09 0 0 0 1.33E-09 -2.49E-09 -4.72E-
09 2.64E-11 0 3.87E-11 0 0 0 0 0 0 -0.017078864
0.021853684 0.016462294 1.58E-07 -6.98E-05 -0.00010444 -0.000750678 -
0.001383265 -9.33E-06 -5.67E-06 1.01E-06 -9.57E-07;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1.01E-05 0 0 0 0 0 0 0 -1.72E-
05];

X=-290:290;
Y= -230:230;
Z= 0:305;
A=0:360;
B=(-pi/2):(pi/2);
C= -pi:pi;
error_values={};
total_error_values={};
color={'b','b--','c','c--','g','g--','y','y--','m','m--','r','r--'};

```

```

list1={}; list2={}; list3={}; list4={}; list5={}; list6={}; list7={};
list8={}; list9={}; list10={}; list11={}; list12={};

for i=0:11

    for j=1:36
        if j==1 || j==2 || j==3 || j==4 || j==5 || j==6
            error_values{i+1,j} = test(5*i+1,j)+ test(5*i+2,j)*X +...
            test(5*i+3,j)*(X.^2)+test(5*i+4,j)*(X.^3)+test(5*i+5,j)*(X.^4)

            %subplot(6,6,j)
            figure(j)
            plot(X,error_values{i+1,j},color{i+1})
            xlabel('X(mm)', 'fontsize',12)
            ylabel('error value(mm)', 'fontsize',12)
            grid on
            title(error{j}, 'fontsize',12)

        elseif j==7 || j==8 || j==9 || j==10 || j==11 || j==12
            error_values{i+1,j} = test(5*i+1,j)+ test(5*i+2,j)*Y +...
            test(5*i+3,j)*(Y.^2)+test(5*i+4,j)*(Y.^3)+test(5*i+5,j)*(Y.^4)
            %subplot(6,6,j)
            figure(j)
            plot(Y,error_values{i+1,j},color{i+1})
            xlabel('Y(mm)', 'fontsize',12)
            ylabel('error value(mm)', 'fontsize',12)
            grid on
            title(error{j}, 'fontsize',12)
        elseif j==13 || j==14 || j==15 || j==16 || j==17 || j==18
            error_values{i+1,j} = test(5*i+1,j)+ test(5*i+2,j)*Z +...
            test(5*i+3,j)*(Z.^2)+test(5*i+4,j)*(Z.^3)+test(5*i+5,j)*(Z.^4)
            %subplot(6,6,j)
            figure(j)
            plot(Z,error_values{i+1,j},color{i+1})
            xlabel('Z(mm)', 'fontsize',12)
            ylabel('error value(mm)', 'fontsize',12)
            grid on
            title(error{j}, 'fontsize',12)
        elseif j==19 || j==20 || j==21 || j==22 || j==23 || j==24
            error_values{i+1,j} = test(5*i+1,j)+ test(5*i+2,j)*A +...
            test(5*i+3,j)*(A.^2)+test(5*i+4,j)*(A.^3)+test(5*i+5,j)*(A.^4)
            %subplot(6,6,j)
            figure(j)
            plot(A,error_values{i+1,j},color{i+1})
            xlabel('A(rad)', 'fontsize',12)
            ylabel('error value(mm)', 'fontsize',12)
            grid on
            title(error{j}, 'fontsize',12)

        elseif j==25 || j==26 || j==27 || j==28 || j==29 || j==30
            error_values{i+1,j} = test(5*i+1,j)+ test(5*i+2,j)*B +...
            test(5*i+3,j)*(B.^2)+test(5*i+4,j)*(B.^3)+test(5*i+5,j)*(B.^4)
        % subplot(6,6,j)
        figure(j)
        plot(B,error_values{i+1,j},color{i+1})
        xlabel('B(rad)', 'fontsize',12)

```

```

ylabel('error value(mm)', 'fontsize', 12)
grid on
title(error{j}, 'fontsize', 12)
else
    error_values{i+1,j} = test(5*i+1,j)+ test(5*i+2,j)*C +...
    test(5*i+3,j)*(C.^2)+test(5*i+4,j)*(C.^3)+test(5*i+5,j)*(C.^4)
% subplot(6,6,j)
figure(j)
plot(C,error_values{i+1,j},color{i+1})
xlabel('C(rad)', 'fontsize', 12)
ylabel('error value(mm)', 'fontsize', 12)
grid on
title(error{j}, 'fontsize', 12)
end

total_error_values{i+1}=error_values;
hold on

if i==0
    list1{j,1}=error{j};
    list1{j,2}=min(error_values{i+1,j});
    list1{j,3}=max(error_values{i+1,j});
elseif i==1
    list2{j,1}=error{j};
    list2{j,2}=min(error_values{i+1,j});
    list2{j,3}=max(error_values{i+1,j});
elseif i==2
    list3{j,1}=error{j};
    list3{j,2}=min(error_values{i+1,j});
    list3{j,3}=max(error_values{i+1,j});
elseif i==3
    list4{j,1}=error{j};
    list4{j,2}=min(error_values{i+1,j});
    list4{j,3}=max(error_values{i+1,j});
elseif i==4
    list5{j,1}=error{j};
    list5{j,2}=min(error_values{i+1,j});
    list5{j,3}=max(error_values{i+1,j});
elseif i==5
    list6{j,1}=error{j};
    list6{j,2}=min(error_values{i+1,j});
    list6{j,3}=max(error_values{i+1,j});
elseif i==6
    list7{j,1}=error{j};
    list7{j,2}=min(error_values{i+1,j});
    list7{j,3}=max(error_values{i+1,j});
elseif i==7
    list8{j,1}=error{j};
    list8{j,2}=min(error_values{i+1,j});
    list8{j,3}=max(error_values{i+1,j});
elseif i==8
    list9{j,1}=error{j};
    list9{j,2}=min(error_values{i+1,j});
    list9{j,3}=max(error_values{i+1,j});
elseif i==9
    list10{j,1}=error{j};
    list10{j,2}=min(error_values{i+1,j});

```

```

        list10{j,3}=max(error_values{i+1,j});
        elseif i==10
list11{j,1}=error{j};
        list11{j,2}=min(error_values{i+1,j});
        list11{j,3}=max(error_values{i+1,j});
        elseif i==11
list12{j,1}=error{j};
        list12{j,2}=min(error_values{i+1,j});
        list12{j,3}=max(error_values{i+1,j});
    end
end
end
end

```

### 3. RANGE OF ERROR AS A STEM FUNCTION (PERFORMANCE OF THE MACHINE)

```
W=[0;200;200;200;200;400;400;600;600;800;800;1000;1000];
```

```
T=[0;5;20;35;10;20;45;90;40;80;75;125];
```

```

error={ 'EXX'          'EAY'          'EXA'          'EAB'
        'EYX'          'EBY'          'EYA'          'EBB'
        'EZX'          'ECY'          'EZA'          'ECB'
        'EAX'          'EXZ'          'EAA'          'EXC'
        'EBX'          'EYZ'          'EBA'          'EYC'
        'ECX'          'EZZ'          'ECA'          'EZC'
        'EXY'          'EAZ'          'EXB'          'EAC'
        'EYY'          'EBZ'          'EYB'          'EBC'
        'EZY'          'ECZ'          'EZB'          'ECC' };

```

```

for j=1:36
    figure(j)
    stem3(W(1),T(1),list1{j,3}-list1{j,2}, 'b')
    grid on
    hold on
    stem3(W(2),T(2),list2{j,3}-list2{j,2}, 'b--')
    stem3(W(3),T(3),list3{j,3}-list3{j,2}, 'c')
    stem3(W(4),T(4),list4{j,3}-list4{j,2}, 'c--')
    stem3(W(5),T(5),list5{j,3}-list5{j,2}, 'g')
    stem3(W(6),T(6),list6{j,3}-list6{j,2}, 'g--')
    stem3(W(7),T(7),list7{j,3}-list7{j,2}, 'y')
    stem3(W(8),T(8),list8{j,3}-list8{j,2}, 'y--')
    stem3(W(9),T(9),list9{j,3}-list9{j,2}, 'm')
    stem3(W(10),T(10),list10{j,3}-list10{j,2}, 'm--')
    stem3(W(11),T(11),list11{j,3}-list11{j,2}, 'r')
    stem3(W(12),T(12),list12{j,3}-list12{j,2}, 'r--')
    hold off
    ylabel('torque (N/m) '); xlabel('weight (N) '); zlabel('error
value (mm) ');
    title(error(j));
end

```

### 4. CORRELATION MATRIX

```
clear all
clc
```

```
test= [Like in previous program]
```

```
W=[0;200;200;200;200;400;400;600;600;800;800;1000;1000];
```

```

T=[0;5;20;35;10;20;45;90;40;80;75;125];

COR_input=[];

m=1;

v=0;w=0;
for n=0:11
    for i=1:36
        for j=1:5
            COR_input(n+1,m)=test(5*n+j,i);
            m=m+1;
        end
    end
    m=1;
end
COR_input(:,181)=W;
COR_input(:,182)=T;
A=[];
p=1;
L1=length(COR_input);
while p<L1

    if sum(COR_input(:,p))==0
        A=[A p];
        % COR_input(:,p)=[];

    end
    p=p+1;
end
for i=1:length(A)
    COR_input(:,A(i))=[];
    A=A-1;
end

COR_input_norm=[];

for i=1:length(COR_input)
    for j=1:12
        COR_input_norm(j,i)=COR_input(j,i)/norm(COR_input(:,i));
    end
end

p=[];
b=1;
correlation1=ones(length(COR_input_norm));

while b<=length(COR_input_norm)
    for n=b:length(COR_input_norm)-1
        p=corrcoef(COR_input_norm(:,b),COR_input_norm(:,n+1));
        % plot(COR_input,W,T)
        correlation1(b,n+1)=p(1,2);
        correlation1(n+1,b)=p(2,1);
    end
    b=b+1;
end

```

end

## 5.MODEL ERROR COEFFICIENT AS A FUCNTION OF WEIGHT OR TORQUE

```
clear all
clc
```

```
test= [Like in previous program
];
```

```
coefficient={
'EXX0' 'EYX0' 'EZX0' 'EAX0' 'EBX0' 'ECX0' 'EXY0' 'EYY0' 'EZY0'
'EAY0' 'EBY0' 'ECY0' 'EXZ0' 'EYZ0' 'EZZ0' 'EAZ0' 'EBZ0' 'ECZ0'
'EXA0' 'EYA0' 'EZA0' 'EAA0' 'EBA0' 'ECA0' 'EXB0' 'EYB0' 'EZB0'
'EAB0' 'EBB0' 'ECB0' 'EXC0' 'EYC0' 'EZC0' 'EAC0' 'EBC0' 'ECC0'
'EXX1' 'EYX1' 'EZX1' 'EAX1' 'EBX1' 'ECX1' 'EXY1' 'EYY1' 'EZY1'
'EAY1' 'EBY1' 'ECY1' 'EXZ1' 'EYZ1' 'EZZ1' 'EAZ1' 'EBZ1' 'ECZ1'
'EXA1' 'EYA1' 'EZA1' 'EAA1' 'EBA1' 'ECA1' 'EXB1' 'EYB1' 'EZB1'
'EAB1' 'EBB1' 'ECB1' 'EXC1' 'EYC1' 'EZC1' 'EAC1' 'EBC1' 'ECC1'
'EXX2' 'EYX2' 'EZX2' 'EAX2' 'EBX2' 'ECX2' 'EXY2' 'EYY2' 'EZY2'
'EAY2' 'EBY2' 'ECY2' 'EXZ2' 'EYZ2' 'EZZ2' 'EAZ2' 'EBZ2' 'ECZ2'
'EXA2' 'EYA2' 'EZA2' 'EAA2' 'EBA2' 'ECA2' 'EXB2' 'EYB2' 'EZB2'
'EAB2' 'EBB2' 'ECB2' 'EXC2' 'EYC2' 'EZC2' 'EAC2' 'EBC2' 'ECC2'
'EXX3' 'EYX3' 'EZX3' 'EAX3' 'EBX3' 'ECX3' 'EXY3' 'EYY3' 'EZY3'
'EAY3' 'EBY3' 'ECY3' 'EXZ3' 'EYZ3' 'EZZ3' 'EAZ3' 'EBZ3' 'ECZ3'
'EXA3' 'EYA3' 'EZA3' 'EAA3' 'EBA3' 'ECA3' 'EXB3' 'EYB3' 'EZB3'
'EAB3' 'EBB3' 'ECB3' 'EXC3' 'EYC3' 'EZC3' 'EAC3' 'EBC3' 'ECC3'
'EXX4' 'EYX4' 'EZX4' 'EAX4' 'EBX4' 'ECX4' 'EXY4' 'EYY4' 'EZY4'
'EAY4' 'EBY4' 'ECY4' 'EXZ4' 'EYZ4' 'EZZ4' 'EAZ4' 'EBZ4' 'ECZ4'
'EXA4' 'EYA4' 'EZA4' 'EAA4' 'EBA4' 'ECA4' 'EXB4' 'EYB4' 'EZB4'
'EAB4' 'EBB4' 'ECB4' 'EXC4' 'EYC4' 'EZC4' 'EAC4' 'EBC4'
'ECC4'};
```

```
W=[0;200;400;600;800;1000];
T=[0;5;20;35;10;20;45;90;40;80;75;125];
```

```
error_coefficient=[];
k=1;
p=1;
u=1;
values={};
```

```
for i=1:36
    for j=0:5
        figure(k)
        plot(W(j+1),test(5*j+1,i),'o')
        values{k,u}=[W(j+1),test(5*j+1,i)];
        xlabel('weight(N)'); title(coefficient{p,i}); ylabel('estimated
error value(mm)')
        grid on
        hold on
        u=u+1;
    end
    k=k+1;
    p=p+1;
```



```

u=1;
for z=0:5
    figure(k)
    plot(W(z+1),test(5*z+2,i),'o')
    values{k,u}=[W(z+1),test(5*z+2,i)];

    xlabel('weight(N)'); title(coefficent{p,i}); ylabel('estimated
error value(mm) ')
    grid on
    hold on
    u=u+1;
end
k=k+1;
p=p+1;
u=1;
for q=0:5
    figure(k)
    plot(W(q+1),test(5*q+3,i),'o')
    values{k,u}=[W(q+1),test(5*q+3,i)];
    xlabel('weight(N)'); title(coefficent{p,i}); ylabel('estimated
error value(mm) ')
    grid on
    hold on
    u=u+1
end
k=k+1;
p=p+1;
u=1;
for n=0:5
    figure(k)
    plot(W(n+1),test(5*n+4,i),'o')
    values{k,u}=[W(n+1),test(5*n+4,i)];
    xlabel('weight(N)'); title(coefficent{p,i}); ylabel('estimated
error value(mm) ')
    grid on
    hold on
    u=u+1;
end
k=k+1;
p=p+1;
u=1;
for m=0:5
    figure(k)
    plot(W(m+1),test(5*m+5,i),'o')
    values{k,u}=[W(m+1),test(5*m+5,i)];
    xlabel('weight(N)'); title(coefficent{p,i}); ylabel('estimated
error value(mm) ')
    grid on
    hold on
    u=u+1;
end
k=k+1;
p=1;
u=1;
end
a=[];
b=[];

```

```

P=[];

for g=1:180
for l=1:6
    a(g,l)=values{g,l}(1);
    b(g,l)=values{g,l}(2);

end

s=fit(a(g,:),b(g,:), 'poly5')
P(g,1)=s.p1;
P(g,2)=s.p2;
P(g,3)=s.p3;
P(g,4)=s.p4;
P(g,5)=s.p5;
P(g,6)=s.p6;
figure(g)
plot(s)
xlabel('weight(N)'); ylabel('estimated error value(mm)')

end

```

## 6.MODEL THE ERROR COEFFICIENT AS A FUNCTION OF WEIGHT AND TORQUE

```

error_coefficient=[];
k=1;
p=1;
u=1;
values={};

for i=1:36
    for j=0:11
        figure(k)
        plot3(W(j+1),T(j+1),test(5*j+1,i), 'o')
        values{k,u}=[W(j+1),T(j+1),test(5*j+1,i)];
        ylabel('torque(N/m)'); title(coefficient{p,i});
        xlabel('weight(N)'); zlabel('estimated error value(mm)')
        grid on
        hold on
        u=u+1;
    end
    k=k+1;
    p=p+1;
    u=1;
    for z=0:11
        figure(k)
        plot3(W(z+1),T(z+1),test(5*z+2,i), 'o')
        values{k,u}=[W(z+1),T(z+1),test(5*z+2,i)];

        ylabel('torque(N/m)'); title(coefficient{p,i});
        xlabel('weight(N)'); zlabel('estimated error value(mm)')
        grid on
        hold on
        u=u+1;
    end
end

```

```

end
k=k+1;
p=p+1;
u=1;
for q=0:11
    figure(k)
    plot3(W(q+1),T(q+1),test(5*q+3,i),'o')
    values{k,u}=[W(q+1),T(q+1),test(5*q+3,i)];
    ylabel('torque (N/m)'); title(coefficent{p,i});
xlabel('weight (N)'); zlabel('estimated error value (mm)')
    grid on
    hold on
    u=u+1
end
k=k+1;
p=p+1;
u=1;
for n=0:11
    figure(k)
    plot3(W(n+1),T(n+1),test(5*n+4,i),'o')
    values{k,u}=[W(n+1),T(n+1),test(5*n+4,i)];
    ylabel('torque (N/m)'); title(coefficent{p,i});
xlabel('weight (N)'); zlabel('estimated error value (mm)')
    grid on
    hold on
    u=u+1;
end
k=k+1;
p=p+1;
u=1;
for m=0:11
    figure(k)
    plot3(W(m+1),T(m+1),test(5*m+5,i),'o')
    values{k,u}=[W(m+1),T(m+1),test(5*m+5,i)];
    ylabel('torque (N/m)'); title(coefficent{p,i});
xlabel('weight (N)'); zlabel('estimated error value (mm)')
    grid on
    hold on
    u=u+1;
end
k=k+1;
p=1;
u=1;
end
a=[];
b=[];
c=[];

for g=1:180
    for l=1:12
        a(g,l)=values{g,l}(1);
        b(g,l)=values{g,l}(2);
        c(g,l)=values{g,l}(3);
    end
end

```

```

s=fit([a(g,:)','b(g,:)'],c(g,:)','poly33')
P(g,1)=s.p00;
P(g,2)=s.p10;
P(g,3)=s.p01;
P(g,4)=s.p20;
P(g,5)=s.p11;
P(g,6)=s.p02;
P(g,7)=s.p30;
P(g,8)=s.p21;
P(g,9)=s.p12;
P(g,10)=s.p03;
figure(g)
plot(s)
ylabel('torque(N/m) '); xlabel('weight(N) '); zlabel('estimated error
value(mm) ')

end

```

**Appendix II-Table B-1: The estimated value of the error coefficients**

	EXX2	EXX3	EYX2	EYX3	EZX2	EZX3	EAX0	EAX1	EAX2	EAX3
TEST1	8.11E-08	6.97E-11	3.02E-07	1.46E-09	-1.76E-07	1.31E-09	-0.00047	1.80E-09	1.14E-09	-3.15E-13
TEST2	-6.51E-08	-2.01E-09	2.12E-07	1.69E-09	7.19E-07	1.93E-09	-0.0004	1.99E-07	2.11E-09	-7.10E-12
TEST3	6.12E-09	2.81E-09	2.01E-07	-1.92E-10	2.79E-07	8.25E-10	-0.00037	1.39E-08	1.04E-09	-1.18E-12
TEST4	4.89E-08	2.70E-09	-6.24E-10	-5.76E-10	-2.40E-08	6.09E-10	-2.73E-05	1.44E-08	2.21E-11	-1.36E-12
TEST5	-1.40E-07	6.06E-10	1.79E-08	8.39E-10	-1.20E-07	-6.20E-10	-0.00021	1.71E-08	-1.42E-09	-1.60E-12
TEST6	-1.62E-08	-1.45E-09	1.90E-07	1.53E-09	1.04E-06	1.17E-09	-0.00037	5.26E-09	1.89E-09	1.43E-12
TEST7	3.05E-08	1.06E-11	3.57E-07	-6.91E-10	2.50E-07	6.17E-10	-0.00048	-2.93E-08	1.54E-09	1.94E-12
TEST8	4.67E-07	5.96E-09	-1.08E-07	-2.20E-09	-3.13E-07	1.90E-09	-0.00047	5.06E-08	8.15E-10	-3.70E-12
TEST9	-9.51E-08	2.54E-10	-1.17E-07	6.36E-10	-8.61E-08	1.64E-10	-0.00015	-7.08E-09	-5.35E-10	-1.86E-12
TEST10	1.43E-08	-1.39E-10	6.20E-08	3.10E-10	-1.06E-06	-2.39E-09	0.000393	2.00E-08	-3.45E-10	-1.75E-12
TEST11	8.11E-08	6.97E-11	3.02E-07	1.46E-09	-1.76E-07	1.31E-09	-0.00047	1.80E-09	1.14E-09	-3.15E-13
TEST12	-2.17E-08	1.87E-09	-5.70E-08	-3.38E-09	8.06E-08	-5.91E-11	-0.00108	-2.30E-08	5.51E-10	4.82E-13
	EBX1	EBX2	EBX3	ECX0	ECX1	ECX2	ECX3	EXY0	EXY2	EXY3
TEST1	4.03E-09	-1.28E-09	2.66E-12	-0.00042	-2.82E-08	1.07E-09	1.09E-13	-0.18927	4.67E-08	-2.70E-09
TEST2	-2.76E-07	-1.34E-09	-1.13E-12	-0.00023	-1.14E-07	1.48E-09	4.57E-12	-0.17752	-1.82E-08	-1.26E-09
TEST3	-2.02E-07	-6.50E-10	2.13E-12	-9.10E-05	-8.72E-08	-3.43E-10	1.94E-12	-0.26355	-5.01E-08	-1.08E-09
TEST4	-7.58E-08	-6.17E-10	2.36E-12	-0.00048	-6.68E-09	-5.56E-10	1.25E-13	-0.19127	1.11E-08	-1.44E-09
TEST5	6.26E-08	5.12E-10	-1.25E-13	-0.00088	3.98E-08	-5.71E-10	-4.77E-13	-0.18826	8.47E-08	-5.07E-09
TEST6	-1.46E-07	-7.16E-10	-1.78E-12	-0.00082	-1.06E-07	1.39E-09	3.64E-12	-0.17771	-1.98E-08	-1.21E-09
TEST7	6.67E-08	-3.53E-10	-1.22E-12	-0.00021	-6.94E-08	-4.64E-10	1.56E-12	-0.19687	2.58E-08	-1.64E-09
TEST8	3.06E-07	-1.49E-09	-1.88E-12	0.001939	-5.86E-08	-2.04E-09	-4.70E-12	-0.28211	-7.44E-09	-7.11E-10
TEST9	-1.03E-07	-4.86E-10	2.49E-12	-0.00019	3.69E-08	1.11E-10	-2.09E-12	-0.22894	2.54E-08	-3.37E-09
TEST10	2.36E-07	3.24E-09	5.61E-12	-0.00063	2.53E-08	2.80E-10	1.91E-12	-0.20762	8.56E-08	8.66E-10
TEST11	4.03E-09	-1.28E-09	2.66E-12	-0.00042	-2.82E-08	1.07E-09	1.09E-13	-0.18927	4.67E-08	-2.70E-09
TEST12	1.16E-07	-1.72E-11	-1.29E-12	0.000464	-2.94E-07	-2.15E-09	4.87E-12	-0.29506	7.08E-09	-2.76E-09
	EYY0	EYY1	EYY2	EYY3	EZY2	EZY3	EXZ1	EXZ2	EXZ3	EYZ2
TEST1	0.043597	2.64E-05	-2.55E-07	6.51E-10	-1.64E-07	4.64E-10	-0.00019	8.59E-07	-1.01E-09	-1.02E-07
TEST2	0.029239	-5.54E-05	-1.16E-07	3.71E-10	-3.67E-07	-1.02E-09	-3.67E-05	3.66E-08	3.10E-10	7.85E-07
TEST3	0.003984	0.000594	1.24E-08	-4.31E-09	-2.30E-07	-1.10E-09	-4.90E-05	3.44E-07	-9.59E-10	9.77E-08
TEST4	0.002171	0.00061	6.82E-08	-4.47E-09	-1.16E-07	-2.10E-10	-0.0001	6.00E-07	-1.07E-09	-4.40E-07
TEST5	0.019832	3.67E-05	-1.26E-07	8.01E-10	-4.17E-08	9.42E-10	6.76E-05	-7.21E-07	2.60E-09	-7.13E-07
TEST6	0.028583	-3.01E-05	-6.35E-08	4.60E-11	-7.56E-07	-1.22E-09	-3.13E-05	2.44E-07	-6.25E-10	9.06E-07
TEST7	0.036363	3.20E-05	-1.33E-07	2.00E-10	-1.77E-07	-1.49E-09	-2.58E-05	-6.90E-08	4.34E-10	4.12E-07
TEST8	0.024382	0.000752	1.02E-07	-4.42E-09	-1.27E-08	-1.12E-09	-0.00038	9.00E-07	-4.23E-10	1.29E-06
TEST9	0.022222	2.12E-05	-6.15E-08	5.37E-10	-1.31E-07	1.37E-09	3.84E-05	-1.41E-07	5.51E-10	-5.36E-07
TEST10	0.029508	4.82E-05	-1.01E-07	-2.31E-10	2.02E-07	-3.77E-10	0.000113	-3.63E-07	7.79E-10	-5.89E-07
TEST11	0.043597	2.64E-05	-2.55E-07	6.51E-10	-1.64E-07	4.64E-10	-0.00019	8.59E-07	-1.01E-09	-1.02E-07
TEST12	0.060245	9.46E-05	-1.66E-07	-5.79E-11	-1.57E-07	-2.05E-09	0.000144	-7.62E-07	1.33E-09	1.51E-06
	EYZ3	EZZ1	EZZ2	EZZ3	EAZ0	EAZ1	EAZ2	EAZ3	ECZ0	ECZ1
TEST1	7.92E-11	7.35E-05	-2.15E-07	2.17E-11	0.000205	-3.62E-06	1.53E-08	-2.03E-11	0.00058	-1.03E-05
TEST2	-3.97E-09	-7.39E-05	-2.76E-07	-2.11E-10	2.23E-05	-6.37E-08	2.42E-09	-2.05E-11	2.85E-05	-7.24E-08
TEST3	-5.79E-10	0.000488	1.22E-06	-6.32E-09	3.28E-05	1.75E-07	-2.06E-09	2.25E-13	-1.15E-05	1.32E-07
TEST4	7.56E-10	0.000488	1.11E-06	-5.43E-09	6.64E-05	6.83E-07	-6.43E-09	1.47E-11	-7.15E-06	-9.12E-07
TEST5	1.46E-09	8.50E-05	-1.48E-08	-6.89E-10	3.21E-05	-4.14E-07	5.67E-09	-1.13E-11	1.33E-06	4.48E-07
TEST6	-3.29E-09	-6.10E-05	-2.88E-07	-1.24E-09	-1.38E-06	-2.42E-07	3.78E-09	-1.95E-11	6.29E-05	-7.61E-07
TEST7	-1.26E-09	3.23E-05	-3.60E-07	6.07E-10	8.40E-06	1.03E-07	3.02E-10	-8.16E-12	-6.66E-05	3.75E-07
TEST8	-2.12E-09	7.10E-05	4.23E-06	-1.04E-08	-0.00066	8.79E-06	-4.68E-08	7.28E-11	-0.0001	1.53E-06
TEST9	1.15E-09	-2.43E-05	3.80E-07	-1.30E-09	7.94E-05	-1.74E-06	1.10E-08	-1.70E-11	5.07E-05	-1.14E-06
TEST10	1.52E-09	-2.27E-05	4.89E-07	-7.25E-10	6.68E-05	-3.08E-08	-5.18E-09	1.84E-11	0.000165	-3.05E-06
TEST11	7.92E-11	7.35E-05	-2.15E-07	2.17E-11	0.000205	-3.62E-06	1.53E-08	-2.03E-11	0.00058	-1.03E-05
TEST12	-2.49E-09	-0.00061	3.22E-06	-4.72E-09	-0.00036	3.16E-06	-1.78E-08	2.64E-11	-0.00053	5.90E-06

	ECZ2	ECZ3	EXB0	EXB1	EXB2	EXB3	EYB1	EYB2	EYB3	EZB1
TEST1	5.85E-08	-1.07E-10	-0.19777	-0.19988	0.035739	0.018371	-0.01426	-0.00014	0.011024	-0.09594
TEST2	7.66E-10	-8.61E-12	-0.17722	-0.19896	0.024252	0.026046	-0.00235	0.001936	-0.00116	-0.07699
TEST3	-6.49E-10	2.57E-12	-0.24917	-0.09166	0.055123	0.011966	-0.0026	-0.00108	0.001407	-0.14577
TEST4	8.79E-09	-1.99E-11	-0.18055	0.01272	0.026662	0.006888	-0.00443	-0.00391	-0.00035	-0.07587
TEST5	-1.84E-08	8.69E-11	-0.17992	-0.18424	0.028219	0.022493	-0.01006	-0.00048	0.006496	-0.08227
TEST6	7.03E-09	-2.37E-11	-0.17159	-0.07224	0.022872	0.013993	-0.00192	0.002282	-0.00196	-0.07271
TEST7	-1.20E-10	-2.54E-12	-0.19841	-0.11003	0.035268	0.015211	-0.00669	-7.85E-05	0.0031	-0.09606
TEST8	-9.80E-09	1.90E-11	-0.30814	-0.17992	0.095542	0.019086	-0.00485	0.000156	0.003719	-0.2173
TEST9	4.03E-09	4.50E-13	-0.21837	-0.1943	0.042621	0.018262	-0.00815	-0.00081	0.004749	-0.12077
TEST10	2.28E-08	-4.73E-11	-0.25907	-0.10638	0.054653	0.027898	0.005499	-0.00462	-0.0113	-0.15086
TEST11	5.85E-08	-1.07E-10	-0.19777	-0.19988	0.035739	0.018371	-0.01426	-0.00014	0.011024	-0.09594
TEST12	-2.64E-08	3.87E-11	-0.27883	0.184076	0.062591	-0.01708	-0.01768	-0.00296	0.021854	-0.18272
	EZB2	EZB3	EAB0	EAB1	EAB2	EAB3	EBB0	EBB1	EBB2	EBB3
TEST1	-0.09119	0.013851	0.000437	-0.00047	-0.00017	9.44E-05	6.40E-06	-4.41E-05	1.37E-05	4.71E-05
TEST2	-0.0818	0.009204	0.000334	-0.00023	-0.00013	1.57E-05	-5.89E-06	-2.62E-05	4.73E-05	6.08E-06
TEST3	-0.03397	0.019256	0.000336	-8.90E-05	-0.00014	7.33E-06	1.13E-05	-8.87E-06	-1.61E-05	3.16E-06
TEST4	0.017602	0.010193	8.72E-05	-0.00048	-3.86E-05	5.90E-05	3.52E-06	2.43E-05	-3.05E-05	-3.70E-05
TEST5	-0.07947	0.013354	0.000289	-0.00095	-0.00013	0.000156	-1.69E-05	-1.90E-05	2.24E-05	1.60E-05
TEST6	-0.02281	0.009057	0.000274	-0.0008	-0.0001	9.37E-05	-1.11E-05	2.19E-05	4.90E-05	-3.57E-05
TEST7	-0.04579	0.010753	0.000402	-0.00021	-0.00015	3.48E-05	-8.72E-06	-3.42E-06	1.64E-05	1.38E-06
TEST8	-0.07533	0.040029	0.000227	0.001953	-5.81E-05	-0.00031	-2.05E-05	-1.54E-05	-7.92E-05	6.15E-06
TEST9	-0.08863	0.016799	0.000225	-0.00024	-8.15E-05	4.53E-05	2.46E-06	-6.86E-05	2.73E-05	4.78E-05
TEST10	-0.0313	0.016356	-0.00034	-0.00061	0.000148	5.20E-05	-0.00016	-5.57E-06	-7.46E-06	-6.94E-05
TEST11	-0.09119	0.013851	0.000437	-0.00047	-0.00017	9.44E-05	6.40E-06	-4.41E-05	1.37E-05	4.71E-05
TEST12	0.097458	0.016462	0.000807	0.000446	-0.00036	1.58E-07	-1.89E-05	6.15E-05	4.56E-05	-6.98E-05
	EBB4	ECB1	ECB2	ECB3	EXC1	EXC2	EXC3	EYC1	EYC2	EYC3
TEST1	2.16E-06	0.000439	-0.00017	-8.80E-05	-0.0033	0.000216	0.00034	0.002404	0.000856	-0.00024
TEST2	-1.30E-06	0.0004	-0.00011	-5.86E-05	0.000879	0.00024	-6.54E-05	0.000567	-0.00094	-6.98E-05
TEST3	2.30E-06	0.000348	-2.69E-05	-4.72E-05	-0.00066	-5.00E-05	7.18E-05	0.002405	-0.00025	-0.00023
TEST4	1.90E-06	0.000118	-0.00019	-3.85E-05	0.004126	9.03E-05	-0.00041	0.001036	-0.00026	-0.0001
TEST5	7.09E-06	0.000317	-0.00035	-5.16E-05	-0.00198	0.000801	0.0002	0.007695	0.000223	-0.00081
TEST6	-1.62E-06	0.000315	-0.00036	-4.08E-05	0.001243	0.000703	-0.00011	0.003115	-0.00086	-0.00033
TEST7	4.69E-06	0.000402	-7.42E-05	-5.77E-05	0.000136	-0.00086	-3.23E-05	0.003202	-0.00063	-0.00033
TEST8	1.27E-05	0.000214	0.000891	-6.49E-05	0.003128	-0.00153	-0.00033	0.014146	-0.00126	-0.0014
TEST9	5.73E-06	0.00022	-7.17E-05	-3.66E-05	-0.00121	0.000541	0.00013	0.005704	0.000174	-0.00059
TEST10	3.04E-06	-0.00032	-0.0003	4.22E-05	0.003502	0.000901	-0.00037	-0.00042	-0.00024	3.11E-05
TEST11	2.16E-06	0.000439	-0.00017	-8.80E-05	-0.0033	0.000216	0.00034	0.002404	0.000856	-0.00024
TEST12	1.01E-05	0.000736	0.00023	-0.0001	0.007532	-0.00074	-0.00075	0.013704	-0.00143	-0.00138

	EZC1	EZC2	EZC3	EAC1	EAC2	EAC3	EBC1	EBC2	EBC3	ECC1	ECC2	ECC3	ECC4
TEST1	8.95E-05	-7.36E-06	6.30E-06	3.19E-06	1.72E-06	-3.72E-07	1.08E-05	-1.94E-07	-1.11E-06	-4.86E-06	-4.31E-06	5.07E-07	-2.91E-05
TEST2	-0.00037	0.000173	3.71E-05	2.43E-07	-4.99E-06	-7.10E-08	-8.74E-06	-4.06E-07	8.36E-07	-3.93E-06	-6.57E-06	-4.40E-07	-1.25E-05
TEST3	0.000242	1.49E-05	2.30E-05	3.54E-06	-1.10E-06	-4.12E-07	1.29E-06	1.83E-06	-1.50E-07	9.25E-07	-4.77E-06	-4.80E-07	-3.14E-05
TEST4	0.000183	-0.00028	-2.39E-05	5.57E-08	-1.08E-06	1.63E-08	-1.59E-05	4.61E-07	1.62E-06	-4.40E-07	-5.82E-06	-2.82E-07	-3.32E-05
TEST5	-0.00062	-6.17E-05	4.93E-05	3.75E-05	-2.92E-06	-3.99E-06	-4.91E-06	-2.32E-06	5.15E-07	-3.65E-06	-5.58E-06	-4.39E-08	-2.68E-05
TEST6	-0.00071	0.000163	7.26E-05	6.60E-06	-4.50E-06	-7.05E-07	-1.39E-05	-2.48E-06	1.46E-06	1.52E-06	-4.37E-06	-5.66E-07	-2.39E-05
TEST7	0.001247	0.000316	-0.00012	2.40E-05	-2.90E-06	-2.44E-06	5.72E-07	2.98E-06	7.59E-08	-2.33E-06	-3.88E-06	-1.95E-07	-2.68E-05
TEST8	0.003086	-0.00012	-0.00022	5.48E-05	-2.66E-06	-5.55E-06	1.08E-06	4.98E-06	-1.00E-07	2.58E-06	-6.28E-06	-1.06E-06	-1.71E-05
TEST9	-0.00016	-3.56E-05	2.48E-05	1.70E-05	8.56E-08	-1.85E-06	5.95E-06	-3.38E-07	-6.52E-07	1.23E-06	-5.19E-06	-4.77E-07	-2.41E-05
TEST10	-0.00082	-0.00017	8.02E-05	-2.62E-05	-2.03E-06	2.64E-06	-2.19E-05	-2.69E-07	2.22E-06	-1.64E-06	-2.39E-06	5.06E-08	-2.90E-05
TEST11	8.95E-05	-7.36E-06	6.30E-06	3.19E-06	1.72E-06	-3.72E-07	1.08E-05	-1.94E-07	-1.11E-06	-4.86E-06	-4.31E-06	5.07E-07	-2.91E-05
TEST12	0.000184	5.54E-05	-9.33E-06	5.55E-05	-2.01E-06	-5.67E-06	-1.07E-05	2.43E-06	1.01E-06	3.14E-06	-5.93E-06	-9.57E-07	-1.72E-05

### Appendix III The equations of nominal modeling, exact modeling and volumetric error (according to figure 3-5 and figure 3-6)

#### Nominal modeling

- **Vector form**

$$\{F\},F P_t^N = \{F\},F O_Z + \{F\},Y O_Y + \{F\},Z O_A + \{F\},A P_t$$

$$\{F\},F P_w^N = \{F\},F O_X + \{F\},X O_B + \{F\},B O_C + \{F\},C P_w$$

$$\{F\},w P_t^N = \{F\},F P_t^N - \{F\},F P_w^N$$

$$\{F\},w P_t^N = \{F\},F O_Z + \{F\},Y O_Y + \{F\},Z O_A + \{F\},A P_t - \{F\},F O_X - \{F\},X O_B - \{F\},B O_C - \{F\},C P_w$$

$$\{F\},w P_t^N = \vec{Z} + \vec{Y} + \vec{A} + \vec{t} - \vec{X} - \vec{B} - \vec{C} - \vec{w}$$

$$\vec{x} = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}, \vec{y} = \begin{bmatrix} 0 \\ y \\ 0 \end{bmatrix}, \vec{z} = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}, \vec{A} = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}, \vec{B} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \vec{C} = \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}, \vec{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}, \vec{w} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

- **HTM:**

$${}^F_t T = {}^F_Z T {}^Y_Y T {}^Z_A T \{F\},A P_t$$

$${}^F_w T = {}^F_X T {}^X_B T {}^B_C T \{F\},C P_w$$

$${}^w_t T = {}^F_t T {}^F_w T^{-1}$$

$${}^F_X T = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^F_Y T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^F_Z T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^F_A T = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^F_B T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^F_C T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#### Exact modeling

- **Vector form:**

$$\{F\},F P_t^V = \{F\},F O_Z + {}^F_Z R [{}^Z\delta_z] + {}^Z_Z R [{}^Z\delta_z] + {}^Z_Z R [{}^Z\delta_z] + {}^Z_Y R [{}^Y\delta_y] + {}^Y_Y R [{}^Y\delta_y] + {}^Y_A R [{}^A\delta_A] + {}^A_A R [{}^A\delta_A]$$

$$\{F\},^F P_W^V = \{F\},^F O_X + \overset{F}{X}R \left[ \overset{X}{\{\}}{\overrightarrow{\delta_x}} + \overset{X}{X}R \left[ \overset{\{X\},\dot{X}}{\overrightarrow{O_B}} + \overset{X}{B}R \left[ \overset{\{B\}}{\overrightarrow{\delta_b}} + \overset{B}{B}R \left[ \overset{\{\dot{B}\},\dot{B}}{\overrightarrow{O_C}} + \overset{B}{C}R \left[ \overset{\{A\}}{\overrightarrow{\delta_c}} + \overset{C}{C}R \left[ \left[ \overset{\{\dot{C}\},\dot{C}}{P_t} \right] \right] \right] \right] \right] \right]$$

$$\{F\},^w P_t^V = \{F\},^F P_t^V - \{F\},^F P_w^V$$

$$\overset{F}{Z}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \overset{\dot{Z}}{Y}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \overset{\dot{Y}}{A}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\overset{F}{X}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \overset{\dot{X}}{B}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \overset{\dot{B}}{C}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\overset{X}{X}R = \text{rot}(z, \varepsilon_z(x)) \text{rot}(y, \varepsilon_y(x)) \text{rot}(x, \varepsilon_x(x))$$

$$\overset{Y}{Y}R = \text{rot}(z, \varepsilon_z(y)) \text{rot}(y, \varepsilon_y(y)) \text{rot}(x, \varepsilon_x(y))$$

$$\overset{Z}{Z}R = \text{rot}(z, \varepsilon_z(z)) \text{rot}(y, \varepsilon_y(z)) \text{rot}(x, \varepsilon_x(z))$$

$$\overset{A}{A}R = \text{rot}(z, \varepsilon_z(a)) \text{rot}(y, \varepsilon_y(a)) \text{rot}(x, \varepsilon_x(a))$$

$$\overset{B}{B}R = \text{rot}(z, \varepsilon_z(b)) \text{rot}(y, \varepsilon_y(b)) \text{rot}(x, \varepsilon_x(b))$$

$$\overset{C}{C}R = \text{rot}(z, \varepsilon_z(c)) \text{rot}(y, \varepsilon_y(c)) \text{rot}(x, \varepsilon_x(c))$$

$$\text{rot}(x, \varepsilon_x(x)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon_x(x)) & -\sin(\varepsilon_x(x)) \\ 0 & \sin(\varepsilon_x(x)) & \cos(\varepsilon_x(x)) \end{bmatrix}$$

$$\text{rot}(x, \varepsilon_x(y)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon_x(y)) & -\sin(\varepsilon_x(y)) \\ 0 & \sin(\varepsilon_x(y)) & \cos(\varepsilon_x(y)) \end{bmatrix}$$

$$\text{rot}(x, \varepsilon_x(z)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon_x(z)) & -\sin(\varepsilon_x(z)) \\ 0 & \sin(\varepsilon_x(z)) & \cos(\varepsilon_x(z)) \end{bmatrix}$$

$$\text{rot}(x, \varepsilon_x(A)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon_x(a)) & -\sin(\varepsilon_x(a)) \\ 0 & \sin(\varepsilon_x(a)) & \cos(\varepsilon_x(a)) \end{bmatrix}$$



$$\text{rot} (x, \varepsilon_x(B)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon_x(b)) & -\sin(\varepsilon_x(b)) \\ 0 & \sin(\varepsilon_x(b)) & \cos(\varepsilon_x(b)) \end{bmatrix}$$

$$\text{rot} (x, \varepsilon_x(C)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon_x(c)) & -\sin(\varepsilon_x(c)) \\ 0 & \sin(\varepsilon_x(c)) & \cos(\varepsilon_x(c)) \end{bmatrix}$$

$$\text{rot} (y, \varepsilon_y(x)) = \begin{bmatrix} \cos(\varepsilon_y(x)) & 0 & \sin(\varepsilon_y(x)) \\ 0 & 1 & 0 \\ -\sin(\varepsilon_y(x)) & 0 & \cos(\varepsilon_y(x)) \end{bmatrix}$$

$$\text{rot} (y, \varepsilon_y(y)) = \begin{bmatrix} \cos(\varepsilon_y(y)) & 0 & \sin(\varepsilon_y(y)) \\ 0 & 1 & 0 \\ -\sin(\varepsilon_y(y)) & 0 & \cos(\varepsilon_y(y)) \end{bmatrix}$$

$$\text{rot} (y, \varepsilon_y(z)) = \begin{bmatrix} \cos(\varepsilon_y(z)) & 0 & \sin(\varepsilon_y(z)) \\ 0 & 1 & 0 \\ -\sin(\varepsilon_y(z)) & 0 & \cos(\varepsilon_y(z)) \end{bmatrix}$$

$$\text{rot} (y, \varepsilon_y(A)) = \begin{bmatrix} \cos(\varepsilon_y(a)) & 0 & \sin(\varepsilon_y(a)) \\ 0 & 1 & 0 \\ -\sin(\varepsilon_y(a)) & 0 & \cos(\varepsilon_y(a)) \end{bmatrix}$$

$$\text{rot} (y, \varepsilon_y(B)) = \begin{bmatrix} \cos(\varepsilon_y(b)) & 0 & \sin(\varepsilon_y(b)) \\ 0 & 1 & 0 \\ -\sin(\varepsilon_y(b)) & 0 & \cos(\varepsilon_y(b)) \end{bmatrix}$$

$$\text{rot} (y, \varepsilon_y(C)) = \begin{bmatrix} \cos(\varepsilon_y(c)) & 0 & \sin(\varepsilon_y(c)) \\ 0 & 1 & 0 \\ -\sin(\varepsilon_y(c)) & 0 & \cos(\varepsilon_y(c)) \end{bmatrix}$$

$$\text{rot}(z, \varepsilon_z(x)) = \begin{bmatrix} \cos(\varepsilon_z(x)) & -\sin(\varepsilon_z(x)) & 0 \\ \sin(\varepsilon_z(x)) & \cos(\varepsilon_z(x)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{rot}(z, \varepsilon_z(y)) = \begin{bmatrix} \cos(\varepsilon_z(y)) & -\sin(\varepsilon_z(y)) & 0 \\ \sin(\varepsilon_z(y)) & \cos(\varepsilon_z(y)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{rot}(z, \varepsilon_z(z)) = \begin{bmatrix} \cos(\varepsilon_z(z)) & -\sin(\varepsilon_z(z)) & 0 \\ \sin(\varepsilon_z(z)) & \cos(\varepsilon_z(z)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{rot}(z, \varepsilon_z(A)) = \begin{bmatrix} \cos(\varepsilon_z(a)) & -\sin(\varepsilon_z(a)) & 0 \\ \sin(\varepsilon_z(a)) & \cos(\varepsilon_z(a)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{rot}(z, \varepsilon_z(B)) = \begin{bmatrix} \cos(\varepsilon_z(b)) & -\sin(\varepsilon_z(b)) & 0 \\ \sin(\varepsilon_z(b)) & \cos(\varepsilon_z(b)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{rot}(z, \varepsilon_z(C)) = \begin{bmatrix} \cos(\varepsilon_z(c)) & -\sin(\varepsilon_z(c)) & 0 \\ \sin(\varepsilon_z(c)) & \cos(\varepsilon_z(c)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• **HTM:**

$${}^F_tT = {}^F_ZT \begin{matrix} Z \\ \dot{Z} \end{matrix}T \begin{matrix} \dot{Z} \\ Y \end{matrix}T \begin{matrix} Y \\ \dot{Y} \end{matrix}T \begin{matrix} \dot{Y} \\ A \end{matrix}T \begin{matrix} A \\ \dot{A} \end{matrix}T \{ \dot{A} \} P_t$$

$${}^F_wT = {}^F_XT \begin{matrix} X \\ \dot{X} \end{matrix}T \begin{matrix} \dot{X} \\ B \end{matrix}T \begin{matrix} B \\ \dot{B} \end{matrix}T \begin{matrix} \dot{B} \\ C \end{matrix}T \begin{matrix} C \\ \dot{C} \end{matrix}T \{ \dot{C} \} P_w$$

$${}^F_ZT = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{\dot{Z}}_YT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{\dot{Y}}_AT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^F_{\dot{X}}T = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{\dot{X}}_BT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{\dot{B}}_{\dot{C}}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^X_{\dot{X}}T = \begin{bmatrix} & & \delta_x(x) \\ & {}^X_{\dot{X}}R & \delta_y(x) \\ & & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^Y_{\dot{Y}}T = \begin{bmatrix} & & \delta_x(y) \\ & {}^Y_{\dot{Y}}R & \delta_y(y) \\ & & \delta_z(y) \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^Z_{\dot{Z}}T = \begin{bmatrix} & & \delta_x(z) \\ & {}^Z_{\dot{Z}}R & \delta_y(z) \\ & & \delta_z(z) \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^A_{\dot{A}}T = \begin{bmatrix} & & \delta_x(a) \\ & {}^A_{\dot{A}}R & \delta_y(a) \\ & & \delta_z(a) \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^B_{\dot{B}}T = \begin{bmatrix} & & \delta_x(b) \\ & {}^B_{\dot{B}}R & \delta_y(b) \\ & & \delta_z(b) \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^C_{\dot{C}}T = \begin{bmatrix} & & \delta_x(c) \\ & {}^C_{\dot{C}}R & \delta_y(c) \\ & & \delta_z(c) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### Volumetric error

1. The nominal position of the tool:

$$\{F\},F P_t^N = \vec{Z} + \vec{y} + \vec{a} + \vec{t}$$

$$\vec{Z} = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}, \vec{y} = \begin{bmatrix} 0 \\ y \\ 0 \end{bmatrix}, \vec{a} = \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix}, \vec{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

2. The nominal position of the workpiece:

$$\{F\},F P_w^N = \vec{x} + \vec{b} + \vec{c} + \vec{w}$$

$$\vec{x} = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}, \vec{b} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \vec{c} = \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}, \vec{w} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

3. The actual position of the tool:

$$\{F\},F P_t^V = {}^F T_Z {}^Z T_{\hat{Z}} {}^{\hat{Z}} T_Y {}^Y T_{\hat{Y}} {}^{\hat{Y}} T_A {}^A T_{\hat{A}} {}^{\hat{A}} P_t$$

4. The actual position of the workpiece:

$$\{F\},F P_w^V = {}^F T_X {}^X T_{\hat{X}} {}^{\hat{X}} T_B {}^B T_{\hat{B}} {}^{\hat{B}} T_C {}^C T_{\hat{C}} {}^{\hat{C}} P_w$$

The volumetric error of the tool can be achieved by following equation:

$$\{F\},F \vec{\delta}_t = \{F\},F P_t^V - \{F\},F P_t^N$$

$$\{F\},F \vec{\delta}_t = \{F\} \vec{\delta}_Z + \{F\} \vec{\varepsilon}_Z \times \{F\} \vec{Z} \vec{P}_t + \{F\} \vec{\delta}_Y + \{F\} \vec{\varepsilon}_Y \times \{F\} \vec{Y} \vec{P}_t + \{F\} \vec{\delta}_A + \{F\} \vec{\varepsilon}_A \times \{F\} \vec{A} \vec{P}_t + \{F\} \vec{\delta}_T + \{F\} \vec{\varepsilon}_T \times \{F\} \vec{T} \vec{P}_t$$

$$\begin{aligned} \{F\},F \vec{\delta}_t &= \{F\} \vec{\delta}_Z + \{F\} \vec{\delta}_Y + \{F\} \vec{\delta}_A + \{F\} \vec{\delta}_T + \{F\} \vec{\varepsilon}_Z \times \begin{bmatrix} t_x \\ t_y + y \\ t_z + a \end{bmatrix} + \{F\} \vec{\varepsilon}_Y \times \begin{bmatrix} t_x \\ t_y \\ t_z + a \end{bmatrix} + \{F\} \vec{\varepsilon}_A \times \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \\ &+ \{F\} \vec{\varepsilon}_T \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

The volumetric error of the workpiece can also be achieved by following equations:

$$\{F\},F \vec{\delta}_w = \{F\},F P_w^V - \{F\},F P_w^N$$

$$\begin{aligned} \{F\},F \vec{\delta}_w &= \{F\} \vec{\delta}_X + \{F\} \vec{\varepsilon}_X \times \{F\} \vec{X} \vec{P}_w + \{F\} \vec{\delta}_B + \{F\} \vec{\varepsilon}_B \times \{F\} \vec{B} \vec{P}_w + \{F\} \vec{\delta}_C + \{F\} \vec{\varepsilon}_C \times \{F\} \vec{C} \vec{P}_w \\ &+ \{F\} \vec{\delta}_W + \{F\} \vec{\varepsilon}_W \times \{F\} \vec{W} \vec{P}_w \end{aligned}$$

$$\begin{aligned} \{F\},F\overrightarrow{\delta_w} &= \{F\}\overrightarrow{\delta_x} + \{F\}\overrightarrow{\delta_B} + \{F\}\overrightarrow{\delta_C} + \{F\}\overrightarrow{\delta_W} + \{F\}\overrightarrow{\varepsilon_X} \times \begin{bmatrix} t_x \\ t_y + b \\ t_z + c \end{bmatrix} + \{F\}\overrightarrow{\varepsilon_B} \times \begin{bmatrix} t_x \\ t_y \\ t_z + c \end{bmatrix} + \{F\}\overrightarrow{\varepsilon_C} \times \\ &\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \{U\}\overrightarrow{\varepsilon_W} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

Finally, the total volumetric error will be achieved by following equation:

$$\begin{aligned} \vec{\delta}_{O_w O_t} &= \{F\},F\overrightarrow{\delta_t} - \{F\},F\overrightarrow{\delta_w} \\ \vec{\delta}_{O_w O_t} &= \{F\}\overrightarrow{\delta_Z} + \{F\}\overrightarrow{\delta_Y} + \{F\}\overrightarrow{\delta_A} + \{F\}\overrightarrow{\delta_T} - \{F\}\overrightarrow{\delta_X} - \{F\}\overrightarrow{\delta_B} - \{F\}\overrightarrow{\delta_C} - \{F\}\overrightarrow{\delta_W} + \\ &\{F\}\overrightarrow{\varepsilon_Z} \times \begin{bmatrix} t_x \\ t_y + y \\ t_z + a \end{bmatrix} + \{F\}\overrightarrow{\varepsilon_Y} \times \begin{bmatrix} t_x \\ t_y \\ t_z + a \end{bmatrix} + \{F\}\overrightarrow{\varepsilon_A} \times \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} - \{F\}\overrightarrow{\varepsilon_X} \times \begin{bmatrix} t_x \\ t_y + b \\ t_z + c \end{bmatrix} - \{F\}\overrightarrow{\varepsilon_B} \times \begin{bmatrix} t_x \\ t_y \\ t_z + c \end{bmatrix} - \\ &\{F\}\overrightarrow{\varepsilon_C} \times \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \end{aligned}$$

The elements of the volumetric error can be written separately:

$$\begin{aligned} \delta_x &= \delta_{ZX} + \delta_{YX} + \delta_{AX} + \delta_{TX} - \delta_{XX} - \delta_{BX} - \delta_{CX} - \delta_{WX} + \varepsilon_{ZY} (t_z + a) - \varepsilon_{ZZ} (t_y + y) + \varepsilon_{YY} \\ &(t_z + a) - \varepsilon_{YZ} (t_y) + \varepsilon_{AY} (t_z) - \varepsilon_{AZ} (t_y) - \varepsilon_{XY} (t_z + c) + \varepsilon_{XZ} (t_y + b) - \varepsilon_{BY} (t_z + c) \\ &+ \varepsilon_{BZ} (t_y) - \varepsilon_{CY} (t_z) + \varepsilon_{CZ} (t_y) \end{aligned}$$

$$\begin{aligned} \delta_y &= \delta_{ZY} + \delta_{YY} + \delta_{AY} + \delta_{TY} - \delta_{XY} - \delta_{BY} - \delta_{CY} - \delta_{WY} - \varepsilon_{ZX} (t_z + a) + \varepsilon_{ZZ} (t_x) - \varepsilon_{YX} \\ &(t_z + a) + \varepsilon_{YZ} (t_x) - \varepsilon_{AX} (t_z) + \varepsilon_{AZ} (t_x) + \varepsilon_{XX} (t_z + c) - \varepsilon_{XZ} (t_x) + \varepsilon_{BX} (t_z + c) - \varepsilon_{BZ} \\ &(t_x) + \varepsilon_{CX} (t_z) - \varepsilon_{CZ} (t_x) \end{aligned}$$

$$\begin{aligned} \delta_z &= \delta_{ZZ} + \delta_{YZ} + \delta_{AZ} + \delta_{TZ} - \delta_{XZ} - \delta_{BZ} - \delta_{CZ} - \delta_{WZ} + \varepsilon_{ZX} (t_y + y) - \varepsilon_{ZY} (t_x) + \varepsilon_{YX} (t_y) \\ &- \varepsilon_{YY} (t_x) + \varepsilon_{AX} (t_y) - \varepsilon_{AY} (t_x) - \varepsilon_{XX} (t_y + b) + \varepsilon_{XY} (t_x) - \varepsilon_{BX} (t_y) + \varepsilon_{BY} (t_x) - \varepsilon_{CX} (t_y) \\ &+ \varepsilon_{CY} (t_x) \end{aligned}$$

$$\varepsilon_A = \varepsilon_{ZX} + \varepsilon_{YX} + \varepsilon_{AX} + \varepsilon_{TX} - \varepsilon_{XX} - \varepsilon_{BX} - \varepsilon_{CX} - \varepsilon_{WX}$$

$$\varepsilon_B = \varepsilon_{ZY} + \varepsilon_{YY} + \varepsilon_{AY} + \varepsilon_{TY} - \varepsilon_{XY} - \varepsilon_{BY} - \varepsilon_{CY} - \varepsilon_{WY}$$

$$\varepsilon_C = \varepsilon_{ZZ} + \varepsilon_{YZ} + \varepsilon_{AZ} + \varepsilon_{TZ} - \varepsilon_{XZ} - \varepsilon_{BZ} - \varepsilon_{CZ} - \varepsilon_{WZ}$$

$$VE = J \delta_p$$

VE is the volumetric error.

J is the Jacobian matrix.

$\delta_p$  is the error parameters matrix.

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \varepsilon_A \\ \varepsilon_B \\ \varepsilon_C \end{bmatrix}_{6 \times 1} = [J]_{6 \times 48} \times \begin{bmatrix} \delta_{ZX} \\ \delta_{ZY} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{WY} \\ \varepsilon_{WZ} \end{bmatrix}_{48 \times 1}$$

**Appendix IV-Table D-1: The elements of the Jacobian matrix**

1	2	3	4	5	6	7	8
1	0	0	0	$(t_z + a)$	$-(t_y + y)$	1	0
0	1	0	$-(t_z + a)$	0	$(t_x)$	0	1
0	0	1	$(t_y + y)$	$-(t_x)$	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0

9	10	11	12	13	14	15	16
0	0	$(t_z + a)$	$-(t_y)$	1	0	0	0
0	$-(t_z + a)$	0	$(t_x)$	0	1	0	$-(t_z)$
1	$(t_y)$	$-(t_x)$	0	0	0	1	$(t_y)$
0	1	0	0	0	0	0	1
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0

17	18	19	20	21	22	23	24
$(t_z)$	$-(t_y)$	1	0	0	0	0	0
0	$(t_x)$	0	1	0	0	0	0
$-(t_x)$	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0
1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1

25	26	27	28	29	30	31	32
-1	0	0	0	$-(t_z + c)$	$(t_y + b)$	-1	0
0	-1	0	$(t_z + c)$	0	$-(t_x)$	0	-1
0	0	-1	$-(t_y + b)$	$(t_x)$	0	0	0
0	0	0	-1	0	0	0	0
0	0	0	0	-1	0	0	0
0	0	0	0	0	-1	0	0

33	34	35	36	37	38	39	40
0	0	$-(t_z + c)$	$(t_y)$	-1	0	0	0
0	$(t_z + c)$	0	$-(t_x)$	0	-1	0	$(t_z)$
-1	$-(t_y)$	$(t_x)$	0	0	0	-1	$-(t_y)$
0	-1	0	0	0	0	0	-1
0	0	-1	0	0	0	0	0
0	0	0	-1	0	0	0	0

41	42	43	44	45	46	47	48
$-(t_z)$	$(t_y)$	-1	0	0	0	0	0
0	$-(t_x)$	0	-1	0	0	0	0
$(t_x)$	0	0	0	-1	0	0	0
0	0	0	0	0	-1	0	0
-1	0	0	0	0	0	-1	0
0	-1	0	0	0	0	0	-1



**Appendix V-Table E-1 : Correlation matrix**

	EXX2	EXX3	EYX2	EYX3	EZX2	EZX3	EAX0	EAX1	EAX2	EAX3
EXX2	1	0.717856	-0.14161	-0.41508	-0.26736	0.41028	-0.13211	0.012295	0.241072	-0.12195
EXX3	0.717856	1	-0.52501	-0.72	-0.34398	0.130696	-0.14718	-0.21429	-0.24034	-0.06038
EYX2	-0.14161	-0.52501	1	0.524197	0.323358	0.297902	-0.11167	0.043897	0.642306	0.276918
EYX3	-0.41508	-0.72	0.524197	1	0.215998	0.109877	0.436601	0.309996	0.142976	-0.15515
EZX2	-0.26736	-0.34398	0.323358	0.215998	1	0.615463	-0.45286	0.275463	0.633031	0.04435
EZX3	0.41028	0.130696	0.297902	0.109877	0.615463	1	-0.52768	0.363789	0.699974	-0.22137
EAX0	-0.13211	-0.14718	-0.11167	0.436601	-0.45286	-0.52768	1	0.097086	-0.44139	-0.21387
EAX1	0.012295	-0.21429	0.043897	0.309996	0.275463	0.363789	0.097086	1	0.333245	-0.89949
EAX2	0.241072	-0.24034	0.642306	0.142976	0.633031	0.699974	-0.44139	0.333245	1	0.000258
EAX3	-0.12195	-0.06038	0.276918	-0.15515	0.04435	-0.22137	-0.21387	-0.89949	0.000258	1
EBX1	0.565107	0.495106	-0.35126	-0.54788	-0.72835	-0.39812	0.043822	-0.37335	-0.3787	0.179047
EBX2	-0.31759	-0.16802	-0.21687	-0.07072	-0.58698	-0.9657	0.587592	-0.21708	-0.55075	0.130907
EBX3	-0.17452	-0.09908	0.036281	0.312435	-0.67397	-0.53931	0.660045	-0.20893	-0.4144	0.018743
ECX0	0.810294	0.770405	-0.43036	-0.69917	-0.14902	0.380543	-0.41325	0.103215	0.147139	-0.27844
ECX1	-0.03671	-0.06245	-0.02538	0.544577	-0.45421	-0.2652	0.812036	-0.04486	-0.50996	-0.13706
ECX2	-0.39187	-0.80542	0.628846	0.941064	0.311684	0.170766	0.32312	0.332096	0.383396	-0.09782
ECX3	-0.57597	-0.59735	0.383115	0.08278	0.480346	-0.11955	-0.25708	0.195651	0.424207	0.125219
EXY0	-0.41529	-0.73501	0.550527	0.839941	0.215404	0.025111	0.442391	0.226533	0.122265	0.001439
EXY2	-0.2057	-0.24054	-0.03885	0.206208	-0.69343	-0.64925	0.416313	-0.28169	-0.63322	0.153128
EXY3	0.417771	0.117025	0.100198	-0.12228	-0.12763	-0.06798	0.355152	0.238766	0.405769	-0.15929
EYY0	0.013039	-0.29099	0.207325	-0.17277	-0.04898	0.004303	-0.59735	-0.16875	0.265965	0.282385
EYY1	0.649419	0.905868	-0.37637	-0.51916	-0.1678	0.230391	0.04174	-0.02731	-0.08226	-0.17053
EYY2	0.401903	0.642354	-0.55351	-0.38154	0.044432	0.090175	0.284483	0.137506	-0.1336	-0.2828
EYY3	-0.56674	-0.80988	0.317645	0.463234	0.069972	-0.21569	-0.1038	-0.03201	-0.00485	0.17485
EZY2	0.221672	0.423765	-0.36326	-0.34318	-0.9364	-0.56296	0.362543	-0.15758	-0.65552	-0.17664
EZY3	-0.26546	-0.18	-0.124	0.563901	-0.37485	-0.18414	0.474111	-0.1082	-0.59187	-0.11333
EXZ1	-0.83165	-0.48712	-0.14285	-0.02778	0.014689	-0.71957	0.122095	-0.15314	-0.36666	0.195403
EXZ2	0.658183	0.300152	0.314388	0.21745	0.035483	0.674744	0.004728	0.083582	0.456714	-0.07554
EXZ3	-0.45976	-0.17268	-0.38115	-0.18766	-0.17914	-0.57881	-0.00092	-0.00339	-0.57859	-0.07361
EYZ2	0.421957	0.218689	-0.0382	-0.51742	0.470058	0.504159	-0.74318	0.197426	0.631983	-0.02942
EYZ3	-0.19755	0.102555	-0.18518	0.157045	-0.72523	-0.61823	0.608565	-0.46842	-0.79982	0.182458
EZZ1	0.138943	0.269994	0.189912	0.312526	-0.05154	0.155517	0.52091	0.030771	-0.07918	-0.09983
EZZ2	0.644211	0.854527	-0.66514	-0.8682	-0.24682	0.078191	-0.35897	-0.07308	-0.1244	-0.14601
EZZ3	-0.66591	-0.92109	0.575397	0.692591	0.099307	-0.22996	0.182326	0.001194	0.062728	0.199689
EAZ0	-0.65107	-0.69805	0.573564	0.78735	0.042613	-0.18015	0.409024	-0.06822	-0.03073	0.181773
EAZ1	0.662543	0.739487	-0.57809	-0.76183	-0.07122	0.127917	-0.21971	0.135776	-0.00429	-0.2513
EAZ2	-0.72487	-0.78114	0.558844	0.78778	0.182011	-0.0875	0.185227	-0.09696	0.00733	0.230663
EAZ3	0.749186	0.846044	-0.60677	-0.80066	-0.42029	-0.06258	-0.06863	-0.04203	-0.17239	-0.16934
ECZ0	0.048658	-0.34548	0.557103	0.764423	-0.20743	0.157983	0.343018	0.015182	0.1254	0.016509
ECZ1	-0.0654	0.315502	-0.55307	-0.67501	0.256216	-0.14124	-0.26016	0.072454	-0.12784	-0.09487
ECZ2	0.10894	-0.31318	0.570891	0.595094	-0.24907	0.13996	0.228489	-0.0906	0.233322	0.1377
ECZ3	-0.18239	0.296073	-0.57097	-0.47124	0.1575	-0.20406	-0.1473	0.053425	-0.42019	-0.14514
EXB0	-0.58881	-0.70516	0.503115	0.753217	0.491594	0.171965	0.188589	0.157767	0.138431	0.0865
EXB1	-0.12046	0.21274	-0.28555	-0.6901	0.102132	-0.25592	-0.38013	-0.34971	-0.02367	0.39594

	EBX1	EBX2	EBX3	ECX0	ECX1	ECX2	ECX3	EXY0	EXY2	EXY3
EXX2	0.565107	-0.31759	-0.17452	0.810294	-0.03671	-0.39187	-0.57597	-0.41529	-0.2057	0.417771
EXX3	0.495106	-0.16802	-0.09908	0.770405	-0.06245	-0.80542	-0.59735	-0.73501	-0.24054	0.117025
EYX2	-0.35126	-0.21687	0.036281	-0.43036	-0.02538	0.628846	0.383115	0.550527	-0.03885	0.100198
EYX3	-0.54788	-0.07072	0.312435	-0.69917	0.544577	0.941064	0.08278	0.839941	0.206208	-0.12228
EZX2	-0.72835	-0.58698	-0.67397	-0.14902	-0.45421	0.311684	0.480346	0.215404	-0.69343	-0.12763
EZX3	-0.39812	-0.9657	-0.53931	0.380543	-0.2652	0.170766	-0.11955	0.025111	-0.64925	-0.06798
EAX0	0.043822	0.587592	0.660045	-0.41325	0.812036	0.32312	-0.25708	0.442391	0.416313	0.355152
EAX1	-0.37335	-0.21708	-0.20893	0.103215	-0.04486	0.332096	0.195651	0.226533	-0.28169	0.238766
EAX2	-0.3787	-0.55075	-0.4144	0.147139	-0.50996	0.383396	0.424207	0.122265	-0.63322	0.405769
EAX3	0.179047	0.130907	0.018743	-0.27844	-0.13706	-0.09782	0.125219	0.001439	0.153128	-0.15929
EBX1	1	0.414152	0.05345	0.479813	0.07162	-0.59935	-0.44522	-0.39583	0.520334	0.140728
EBX2	0.414152	1	0.506463	-0.34213	0.236883	-0.09469	0.188156	0.018146	0.587572	0.280814
EBX3	0.05345	0.506463	1	-0.40224	0.565696	0.286677	-0.14421	0.178475	0.474908	0.191527
ECX0	0.479813	-0.34213	-0.40224	1	-0.34209	-0.67287	-0.44086	-0.76801	-0.38023	0.223685
ECX1	0.07162	0.236883	0.565696	-0.34209	1	0.317582	-0.62493	0.497104	0.539864	-0.08989
ECX2	-0.59935	-0.09469	0.286677	-0.67287	0.317582	1	0.299033	0.807956	0.067167	0.067629
ECX3	-0.44522	0.188156	-0.14421	-0.44086	-0.62493	0.299033	1	0.168176	-0.23105	0.167724
EXY0	-0.39583	0.018146	0.178475	-0.76801	0.497104	0.807956	0.168176	1	0.354613	-0.10154
EXY2	0.520334	0.587572	0.474908	-0.38023	0.539864	0.067167	-0.23105	0.354613	1	-0.30136
EXY3	0.140728	0.280814	0.191527	0.223685	-0.08989	0.067629	0.167724	-0.10154	-0.30136	1
EYY0	0.327026	-0.00944	-0.20309	0.113831	-0.529	-0.01049	0.322402	-0.11747	0.26051	-0.17057
EYY1	0.200444	-0.22022	-0.02348	0.606692	0.018924	-0.57879	-0.4794	-0.53523	-0.43823	0.316894
EYY2	0.040431	-0.04357	-0.16437	0.44815	0.107032	-0.44546	-0.36249	-0.3633	-0.48694	0.400745
EYY3	-0.07487	0.179314	0.004436	-0.51039	0.018963	0.482969	0.333342	0.465698	0.528556	-0.43747
EZY2	0.670656	0.519428	0.534676	0.236547	0.385329	-0.46646	-0.46302	-0.29486	0.609202	0.04569
EZY3	-0.07599	0.048562	0.545728	-0.39794	0.804017	0.341997	-0.53135	0.41093	0.582423	-0.52982
EXZ1	-0.15834	0.660511	0.148478	-0.54231	-0.16972	0.000595	0.615276	0.041343	0.253193	-0.1553
EXZ2	-0.07761	-0.61802	0.112673	0.274673	0.20802	0.256363	-0.43982	0.120602	-0.30061	0.26683
EXZ3	0.285346	0.514794	-0.19839	-0.12017	-0.01917	-0.32779	0.124232	-0.08485	0.49008	-0.43774
EYZ2	0.093131	-0.40947	-0.76698	0.633796	-0.84982	-0.3263	0.29649	-0.48943	-0.61555	0.235797
EYZ3	0.262993	0.484623	0.774775	-0.36149	0.750171	-0.04122	-0.47381	0.145217	0.698307	-0.2573
EZZ1	-0.28279	-0.15083	0.369009	-0.17168	0.604768	0.17593	-0.39354	0.266643	-0.14988	0.161613
EZZ2	0.533991	-0.08203	-0.28225	0.8713	-0.432	-0.84894	-0.29646	-0.88655	-0.30725	0.189161
EZZ3	-0.28864	0.21938	0.227193	-0.77936	0.249547	0.714541	0.382016	0.7578	0.511155	-0.27309
EAZ0	-0.53958	0.132369	0.588015	-0.8882	0.464138	0.772385	0.24845	0.750295	0.329813	-0.19028
EAZ1	0.511161	-0.05402	-0.51862	0.842183	-0.34604	-0.7635	-0.27009	-0.67898	-0.34933	0.326585
EAZ2	-0.57784	0.007331	0.404206	-0.85629	0.335358	0.769707	0.282409	0.701884	0.306263	-0.40481
EAZ3	0.704378	0.11646	-0.17107	0.820083	-0.20399	-0.80154	-0.39139	-0.71892	-0.13059	0.400608
ECZ0	-0.13064	-0.13904	0.544817	-0.39733	0.584661	0.725421	-0.24985	0.61002	0.36396	-0.03577
ECZ1	0.078124	0.139954	-0.5747	0.370432	-0.50209	-0.66621	0.220236	-0.54793	-0.38704	0.050097
ECZ2	-0.06962	-0.11794	0.595459	-0.33027	0.384381	0.655511	-0.12267	0.491912	0.309332	0.100242
ECZ3	0.098256	0.146054	-0.52789	0.22445	-0.18604	-0.62435	-0.01301	-0.39014	-0.11934	-0.30633
EXB0	-0.64455	-0.20109	-0.00869	-0.78961	0.308264	0.714714	0.268821	0.901488	0.12969	-0.35512
EXB1	0.105411	0.210663	-0.17296	0.085994	-0.69921	-0.5239	0.507155	-0.45797	-0.23274	0.113853

	EYY0	EYY1	EYY2	EYY3	EZY2	EZY3	EXZ1	EXZ2	EXZ3	EY22
EXX2	0.013039	0.649419	0.401903	-0.56674	0.221672	-0.26546	-0.83165	0.658183	-0.45976	0.421957
EXX3	-0.29099	0.905868	0.642354	-0.80988	0.423765	-0.18	-0.48712	0.300152	-0.17268	0.218689
EYX2	0.207325	-0.37637	-0.55351	0.317645	-0.36326	-0.124	-0.14285	0.314388	-0.38115	-0.0382
EYX3	-0.17277	-0.51916	-0.38154	0.463234	-0.34318	0.563901	-0.02778	0.21745	-0.18766	-0.51742
EZX2	-0.04898	-0.1678	0.044432	0.069972	-0.9364	-0.37485	0.014689	0.035483	-0.17914	0.470058
EZX3	0.004303	0.230391	0.090175	-0.21569	-0.56296	-0.18414	-0.71957	0.674744	-0.57881	0.504159
EAX0	-0.59735	0.04174	0.284483	-0.1038	0.362543	0.474111	0.122095	0.004728	-0.00092	-0.74318
EAX1	-0.16875	-0.02731	0.137506	-0.03201	-0.15758	-0.1082	-0.15314	0.083582	-0.00339	0.197426
EAX2	0.265965	-0.08226	-0.1336	-0.00485	-0.65552	-0.59187	-0.36666	0.456714	-0.57859	0.631983
EAX3	0.282385	-0.17053	-0.2828	0.17485	-0.17664	-0.11333	0.195403	-0.07554	-0.07361	-0.02942
EBX1	0.327026	0.200444	0.040431	-0.07487	0.670656	-0.07599	-0.15834	-0.07761	0.285346	0.093131
EBX2	-0.00944	-0.22022	-0.04357	0.179314	0.519428	0.048562	0.660511	-0.61802	0.514794	-0.40947
EBX3	-0.20309	-0.02348	-0.16437	0.004436	0.534676	0.545728	0.148478	0.112673	-0.19839	-0.76698
ECX0	0.113831	0.606692	0.44815	-0.51039	0.236547	-0.39794	-0.54231	0.274673	-0.12017	0.633796
ECX1	-0.529	0.018924	0.107032	0.018963	0.385329	0.804017	-0.16972	0.20802	-0.01917	-0.84982
ECX2	-0.01049	-0.57879	-0.44546	0.482969	-0.46646	0.341997	0.000595	0.256363	-0.32779	-0.3263
ECX3	0.322402	-0.4794	-0.36249	0.333342	-0.46302	-0.53135	0.615276	-0.43982	0.124232	0.29649
EXY0	-0.11747	-0.53523	-0.3633	0.465698	-0.29486	0.41093	0.041343	0.120602	-0.08485	-0.48943
EXY2	0.26051	-0.43823	-0.48694	0.528556	0.609202	0.582423	0.253193	-0.30061	0.49008	-0.61555
EXY3	-0.17057	0.316894	0.400745	-0.43747	0.04569	-0.52982	-0.1553	0.26683	-0.43774	0.235797
EYY0	1	-0.58272	-0.74756	0.647371	-0.02988	-0.22412	0.12643	-0.19006	0.210994	0.445724
EYY1	-0.58272	1	0.802599	-0.97733	0.251546	-0.21917	-0.52528	0.448218	-0.4044	0.132545
EYY2	-0.74756	0.802599	1	-0.84402	0.042214	-0.22695	-0.24128	0.156995	-0.17429	0.120063
EYY3	0.647371	-0.97733	-0.84402	1	-0.14751	0.302439	0.446422	-0.42957	0.462092	-0.13998
EZY2	-0.02988	0.251546	0.042214	-0.14751	1	0.295033	0.030377	-0.14782	0.311861	-0.42596
EZY3	-0.22412	-0.21917	-0.22695	0.302439	0.295033	1	-0.04179	0.098105	0.090252	-0.812
EXZ1	0.12643	-0.52528	-0.24128	0.446422	0.030377	-0.04179	1	-0.9065	0.662275	-0.21227
EXZ2	-0.19006	0.448218	0.156995	-0.42957	-0.14782	0.098105	-0.9065	1	-0.89828	0.051561
EXZ3	0.210994	-0.4044	-0.17429	0.462092	0.311861	0.090252	0.662275	-0.89828	1	-0.09371
EY22	0.445724	0.132545	0.120063	-0.13998	-0.42596	-0.812	-0.21227	0.051561	-0.09371	1
EY23	-0.30488	0.046047	-0.06883	0.020154	0.670736	0.746522	0.16316	-0.07822	0.156716	-0.89939
EZZ1	-0.84786	0.562017	0.460585	-0.59825	0.079665	0.303709	-0.40289	0.535241	-0.51251	-0.53592
EZZ2	0.048325	0.683331	0.527279	-0.60731	0.312582	-0.42866	-0.26355	0.042555	0.019108	0.553811
EZZ3	0.348428	-0.91203	-0.77342	0.874183	-0.16942	0.365977	0.424882	-0.27391	0.237298	-0.41411
EAZ0	-0.12646	-0.5148	-0.51198	0.44304	-0.12786	0.544202	0.275562	0.040538	-0.16126	-0.72198
EAZ1	-0.08989	0.6399	0.680688	-0.59634	0.177785	-0.55658	-0.28411	-0.01106	0.116927	0.620396
EAZ2	0.063406	-0.68179	-0.65714	0.637465	-0.26468	0.557627	0.320077	-0.05417	-0.04123	-0.59044
EAZ3	-0.06478	0.717874	0.611692	-0.65407	0.465859	-0.41207	-0.31982	0.088355	0.017301	0.414723
ECZ0	0.018292	-0.25715	-0.46805	0.280699	0.013676	0.606359	-0.41366	0.595643	-0.47761	-0.51753
ECZ1	-0.12456	0.260726	0.535005	-0.29184	-0.0554	-0.57401	0.399466	-0.59814	0.5006	0.483478
ECZ2	0.187549	-0.24208	-0.52173	0.249	0.030016	0.437596	-0.38379	0.625644	-0.59507	-0.38709
ECZ3	-0.24508	0.19413	0.44355	-0.16477	0.066203	-0.20525	0.380298	-0.65623	0.712323	0.185605
EXB0	-0.16639	-0.49896	-0.32817	0.424988	-0.50164	0.368165	0.135929	0.037079	-0.06152	-0.40287
EXB1	0.210117	0.162357	0.13223	-0.22253	-0.06588	-0.60045	0.448249	-0.38783	0.115953	0.399878

	EY23	EZZ1	EZZ2	EZZ3	EAZ0	EAZ1	EAZ2	EAZ3	ECZ0	ECZ1
EXX2	-0.19755	0.138943	0.644211	-0.66591	-0.65107	0.662543	-0.72487	0.749186	0.048658	-0.0654
EXX3	0.102555	0.269994	0.854527	-0.92109	-0.69805	0.739487	-0.78114	0.846044	-0.34548	0.315502
EYX2	-0.18518	0.189912	-0.66514	0.575397	0.573564	-0.57809	0.558844	-0.60677	0.557103	-0.55307
EYX3	0.157045	0.312526	-0.8682	0.692591	0.78735	-0.76183	0.78778	-0.80066	0.764423	-0.67501
EZX2	-0.72523	-0.05154	-0.24682	0.099307	0.042613	-0.07122	0.182011	-0.42029	-0.20743	0.256216
EZX3	-0.61823	0.155517	0.078191	-0.22996	-0.18015	0.127917	-0.0875	-0.06258	0.157983	-0.14124
EAX0	0.608565	0.52091	-0.35897	0.182326	0.409024	-0.21971	0.185227	-0.06863	0.343018	-0.26016
EAX1	-0.46842	0.030771	-0.07308	0.001194	-0.06822	0.135776	-0.09696	-0.04203	0.015182	0.072454
EAX2	-0.79982	-0.07918	-0.1244	0.062728	-0.03073	-0.00429	0.00733	-0.17239	0.1254	-0.12784
EAX3	0.182458	-0.09983	-0.14601	0.199689	0.181773	-0.2513	0.230663	-0.16934	0.016509	-0.09487
EBX1	0.262993	-0.28279	0.533991	-0.28864	-0.53958	0.511161	-0.57784	0.704378	-0.13064	0.078124
EBX2	0.484623	-0.15083	-0.08203	0.21938	0.132369	-0.05402	0.007331	0.11646	-0.13904	0.139954
EBX3	0.774775	0.369009	-0.28225	0.227193	0.588015	-0.51862	0.404206	-0.17107	0.544817	-0.5747
ECX0	-0.36149	-0.17168	0.8713	-0.77936	-0.8882	0.842183	-0.85629	0.820083	-0.39733	0.370432
ECX1	0.750171	0.604768	-0.432	0.249547	0.464138	-0.34604	0.335358	-0.20399	0.584661	-0.50209
ECX2	-0.04122	0.17593	-0.84894	0.714541	0.772385	-0.7635	0.769707	-0.80154	0.725421	-0.66621
ECX3	-0.47381	-0.39354	-0.29646	0.382016	0.24845	-0.27009	0.282409	-0.39139	-0.24985	0.220236
EXY0	0.145217	0.266643	-0.88655	0.7578	0.750295	-0.67898	0.701884	-0.71892	0.61002	-0.54793
EXY2	0.698307	-0.14988	-0.30725	0.511155	0.329813	-0.34933	0.306263	-0.13059	0.36396	-0.38704
EXY3	-0.2573	0.161613	0.189161	-0.27309	-0.19028	0.326585	-0.40481	0.400608	-0.03577	0.050097
EYY0	-0.30488	-0.84786	0.048325	0.348428	-0.12646	-0.08989	0.063406	-0.06478	0.018292	-0.12456
EYY1	0.046047	0.562017	0.683331	-0.91203	-0.5148	0.6399	-0.68179	0.717874	-0.25715	0.260726
EYY2	-0.06883	0.460585	0.527279	-0.77342	-0.51198	0.680688	-0.65714	0.611692	-0.46805	0.535005
EYY3	0.020154	-0.59825	-0.60731	0.874183	0.44304	-0.59634	0.637465	-0.65407	0.280699	-0.29184
EZY2	0.670736	0.079665	0.312582	-0.16942	-0.12786	0.177785	-0.26468	0.465859	0.013676	-0.0554
EZY3	0.746522	0.303709	-0.42866	0.365977	0.544202	-0.55658	0.557627	-0.41207	0.606359	-0.57401
EXZ1	0.16316	-0.40289	-0.26355	0.424882	0.275562	-0.28411	0.320077	-0.31982	-0.41366	0.399466
EXZ2	-0.07822	0.535241	0.042555	-0.27391	0.040538	-0.01106	-0.05417	0.088355	0.595643	-0.59814
EXZ3	0.156716	-0.51251	0.019108	0.237298	-0.16126	0.116927	-0.04123	0.017301	-0.47761	0.5006
EYZ2	-0.89939	-0.53592	0.553811	-0.41411	-0.72198	0.620396	-0.59044	0.414723	-0.51753	0.483478
EYZ3	1	0.405342	-0.21017	0.169046	0.448039	-0.38592	0.326449	-0.09164	0.366583	-0.37952
EZZ1	0.405342	1	-0.20952	-0.20586	0.346429	-0.1531	0.127947	-0.06869	0.381479	-0.31137
EZZ2	-0.21017	-0.20952	1	-0.89453	-0.90342	0.875278	-0.90592	0.918606	-0.59784	0.543817
EZZ3	0.169046	-0.20586	-0.89453	1	0.771036	-0.82317	0.845614	-0.8426	0.481218	-0.47131
EAZ0	0.448039	0.346429	-0.90342	0.771036	1	-0.96438	0.945537	-0.86826	0.66522	-0.65301
EAZ1	-0.38592	-0.1531	0.875278	-0.82317	-0.96438	1	-0.98602	0.91265	-0.67513	0.684329
EAZ2	0.326449	0.127947	-0.90592	0.845614	0.945537	-0.98602	1	-0.96171	0.611846	-0.6038
EAZ3	-0.09164	-0.06869	0.918606	-0.8426	-0.86826	0.91265	-0.96171	1	-0.49893	0.468712
ECZ0	0.366583	0.381479	-0.59784	0.481218	0.66522	-0.67513	0.611846	-0.49893	1	-0.98447
ECZ1	-0.37952	-0.31137	0.543817	-0.47131	-0.65301	0.684329	-0.6038	0.468712	-0.98447	1
ECZ2	0.299674	0.255937	-0.48936	0.436691	0.620439	-0.65082	0.551393	-0.41444	0.939402	-0.97627
ECZ3	-0.1058	-0.17065	0.385339	-0.3478	-0.51398	0.542183	-0.43274	0.323063	-0.81536	0.869468
EXB0	0.048064	0.258925	-0.85565	0.690905	0.746091	-0.70448	0.765669	-0.8293	0.418626	-0.36764
EXB1	-0.21038	-0.36493	0.449986	-0.31855	-0.31714	0.29999	-0.3276	0.327437	-0.68716	0.578218

	EC22	EC23	EXB0	EXB1	EXB2	EXB3	EYB1	EYB2	EYB3	EZB1
EXX2	0.10894	-0.18239	-0.58881	-0.12046	0.730618	0.041639	0.043938	0.042861	0.010015	-0.60497
EXX3	-0.31318	0.296073	-0.70516	0.21274	0.787955	-0.30362	-0.07471	-0.33072	0.183997	-0.7187
EYX2	0.570891	-0.57097	0.503115	-0.28555	-0.49522	0.251787	-0.03852	0.409634	-0.05736	0.530459
EYX3	0.595094	-0.47124	0.753217	-0.6901	-0.71681	0.705287	0.208979	0.461486	-0.38691	0.762787
EZX2	-0.24907	0.1575	0.491594	0.102132	-0.45839	-0.20241	-0.05241	0.714572	0.086597	0.450877
EZX3	0.13996	-0.20406	0.171965	-0.25592	-0.01949	-0.02896	-0.27284	0.719675	0.269206	0.122264
EAX0	0.228489	-0.1473	0.188589	-0.38013	-0.19517	0.681488	0.740429	-0.34913	-0.84394	0.236891
EAX1	-0.0906	0.053425	0.157767	-0.34971	-0.11554	0.433949	0.359315	0.383142	-0.33633	0.144603
EAX2	0.233322	-0.42019	0.138431	-0.02367	-0.10376	-0.04509	0.040046	0.595995	0.002194	0.132447
EAX3	0.1377	-0.14514	0.0865	0.39594	-0.13581	-0.443	-0.31766	-0.14775	0.289043	0.099683
EBX1	-0.06962	0.098256	-0.64455	0.105411	0.668861	-0.04187	-0.06915	-0.42739	0.102605	-0.64258
EBX2	-0.11794	0.146054	-0.20109	0.210663	0.05423	0.14135	0.452946	-0.65643	-0.43408	-0.14574
EBX3	0.595459	-0.52789	-0.00869	-0.17296	-0.08197	0.301407	0.218079	-0.6145	-0.31192	0.061116
ECX0	-0.33027	0.22445	-0.78961	0.085994	0.879879	-0.23307	-0.11542	0.034389	0.26126	-0.82522
ECX1	0.384381	-0.18604	0.308264	-0.69921	-0.22229	0.764817	0.338724	-0.09523	-0.52052	0.326555
ECX2	0.655511	-0.62435	0.714714	-0.5239	-0.71516	0.56464	0.221157	0.452068	-0.36876	0.733219
ECX3	-0.12267	-0.01301	0.268821	0.507155	-0.43457	-0.34428	0.055851	0.029437	0.019916	0.294422
EXY0	0.491912	-0.39014	0.901488	-0.45797	-0.86478	0.56852	0.194928	0.303921	-0.3996	0.914231
EXY2	0.309332	-0.11934	0.12969	-0.23274	-0.15763	0.325709	-0.13549	-0.38524	-0.0095	0.150156
EXY3	0.100242	-0.30633	-0.35512	0.113853	0.31992	0.177348	0.747674	-0.20712	-0.65709	-0.30543
EYY0	0.187549	-0.24508	-0.16639	0.210117	0.101829	-0.33663	-0.58953	0.100174	0.609297	-0.18089
EYY1	-0.24208	0.19413	-0.49896	0.162357	0.584905	-0.17235	0.196221	-0.27902	-0.10068	-0.49743
EYY2	-0.52173	0.44355	-0.32817	0.13223	0.400122	-0.00861	0.552028	-0.14796	-0.44973	-0.33626
EYY3	0.249	-0.16477	0.424988	-0.22253	-0.48841	0.171771	-0.32881	0.282906	0.224804	0.414349
EZY2	0.030016	0.066203	-0.50164	-0.06588	0.472779	0.147771	0.019475	-0.71481	-0.03035	-0.46689
EZY3	0.437596	-0.20525	0.368165	-0.60045	-0.31771	0.486222	-0.16073	-0.04417	-0.02811	0.364901
EXZ1	-0.38379	0.380298	0.135929	0.448249	-0.33348	-0.24936	0.08502	-0.36507	-0.04088	0.162738
EXZ2	0.625644	-0.65623	0.037079	-0.38783	0.111776	0.229718	-0.01077	0.270479	-0.0621	0.035394
EXZ3	-0.59507	0.712323	-0.06152	0.115953	-0.00777	-0.02902	-0.09873	-0.14866	0.130402	-0.08089
EYZ2	-0.38709	0.185605	-0.40287	0.399878	0.422619	-0.48777	-0.16944	0.369735	0.34305	-0.44437
EYZ3	0.299674	-0.1058	0.048064	-0.21038	-0.07397	0.255225	-0.00347	-0.60044	-0.126	0.086951
EZZ1	0.255937	-0.17065	0.258925	-0.36493	-0.16573	0.402712	0.388582	-0.063	-0.47626	0.287817
EZZ2	-0.48936	0.385339	-0.85565	0.449986	0.876625	-0.50395	-0.12033	-0.30664	0.303104	-0.87909
EZZ3	0.436691	-0.3478	0.690905	-0.31855	-0.75435	0.357587	-0.08561	0.215454	-0.07513	0.708956
EAZ0	0.620439	-0.51398	0.746091	-0.31714	-0.80671	0.35409	0.031246	0.000586	-0.21077	0.789397
EAZ1	-0.65082	0.542183	-0.70448	0.29999	0.770755	-0.25361	0.176372	-0.08199	-0.00229	-0.73619
EAZ2	0.551393	-0.43274	0.765669	-0.3276	-0.82124	0.265518	-0.17944	0.188747	0.007231	0.786891
EAZ3	-0.41444	0.323063	-0.8293	0.327437	0.870155	-0.25547	0.145866	-0.39506	0.002977	-0.83691
ECZ0	0.939402	-0.81536	0.418626	-0.68716	-0.35175	0.605425	-0.03865	0.211497	-0.15456	0.441244
ECZ1	-0.97627	0.869468	-0.36764	0.578218	0.318639	-0.47722	0.148958	-0.12147	0.038822	-0.39348
ECZ2	1	-0.95388	0.299126	-0.44974	-0.27513	0.378979	-0.11203	0.050214	-0.05247	0.333662
ECZ3	-0.95388	1	-0.20116	0.285554	0.200029	-0.24647	0.038915	-0.03109	0.080204	-0.24029
EXB0	0.299126	-0.20116	1	-0.28154	-0.97184	0.2889	-0.02366	0.386206	-0.15672	0.995929
EXB1	-0.44974	0.285554	-0.28154	1	0.13965	-0.8942	-0.17558	-0.51783	0.322844	-0.26826

	EZB2	EZB3	EAB0	EAB1	EAB2	EAB3	EBB0	EBB1	EBB2	EBB3
EXX2	-0.11559	0.816667	-0.06527	0.815159	0.147013	-0.85033	-0.01081	-0.00067	-0.79865	0.055436
EXX3	0.206521	0.821381	0.027666	0.766344	-0.0107	-0.74068	0.105127	0.198223	-0.8469	-0.07451
EYX2	-0.29581	-0.50134	0.176604	-0.4236	-0.13853	0.409711	0.19369	-0.17138	0.310793	0.285016
EYX3	-0.68285	-0.53643	-0.28897	-0.70107	0.318985	0.602241	0.069487	-0.61373	0.385491	0.493038
EZX2	0.086764	-0.39812	0.485029	-0.14074	-0.46386	0.139171	0.62472	0.216386	0.503112	0.058003
EZX3	-0.28528	0.156449	0.503693	0.377801	-0.41127	-0.35706	0.776961	-0.17764	-0.08039	0.496655
EAX0	-0.33333	-0.10769	-0.96236	-0.40165	0.944987	0.210222	-0.58079	-0.31985	-0.28035	-0.07823
EAX1	-0.31091	-0.01056	-0.14066	0.112394	0.168232	-0.21065	-0.00603	-0.18607	0.053746	0.071004
EAX2	-0.02618	-0.1273	0.350883	0.165089	-0.28338	-0.18905	0.319599	0.106034	0.186737	0.049753
EAX3	0.360584	-0.25857	0.252457	-0.27944	-0.27533	0.361612	0.070292	0.333054	0.193488	-0.15551
EBX1	0.116154	0.606503	-0.19233	0.476194	0.195739	-0.45024	-0.54414	0.155379	-0.50339	-0.25694
EBX2	0.255301	-0.12428	-0.61157	-0.33151	0.530081	0.266048	-0.89786	0.210978	0.036804	-0.58396
EBX3	-0.15966	-0.13944	-0.57391	-0.40377	0.540495	0.358632	-0.44004	-0.36602	-0.17206	0.057277
ECX0	0.081948	0.886753	0.220278	0.999262	-0.16336	-0.9616	0.094449	0.091034	-0.6035	-0.00253
ECX1	-0.68789	-0.00274	-0.69874	-0.34974	0.720858	0.225633	-0.19363	-0.67932	-0.32207	0.450562
ECX2	-0.5146	-0.61474	-0.21509	-0.66718	0.245174	0.574549	0.036579	-0.4669	0.482839	0.341701
ECX3	0.522645	-0.66404	0.263428	-0.4243	-0.3272	0.450729	-0.10841	0.535268	0.711437	-0.52194
EXY0	-0.44808	-0.71796	-0.3054	-0.76492	0.314979	0.666906	0.007581	-0.31347	0.382365	0.24483
EXY2	-0.22853	-0.1569	-0.33646	-0.39556	0.302474	0.418548	-0.50751	-0.26293	0.069547	0.067288
EXY3	0.167622	0.211567	-0.54646	0.257839	0.564297	-0.42099	-0.54621	0.246345	-0.39348	-0.51238
EYY0	0.186589	-0.0696	0.54302	0.105029	-0.53817	0.051242	-0.09211	0.13076	0.471622	-0.05478
EYY1	0.169586	0.651986	-0.14505	0.612644	0.163276	-0.65972	0.150863	0.202801	-0.87311	-0.10844
EYY2	0.160071	0.491153	-0.39227	0.463875	0.399677	-0.59319	-0.00842	0.229438	-0.65646	-0.27612
EYY3	-0.23773	-0.53222	0.209277	-0.52294	-0.21965	0.597946	-0.11384	-0.27886	0.808072	0.214696
EZY2	-0.05303	0.408535	-0.38001	0.226545	0.351666	-0.20735	-0.51177	-0.18169	-0.52272	-0.03797
EZY3	-0.6191	-0.12137	-0.2866	-0.42273	0.292864	0.42255	0.07484	-0.74903	0.014744	0.632966
EXZ1	0.460856	-0.55545	-0.03785	-0.53906	-0.06875	0.576413	-0.37484	0.327649	0.643141	-0.48581
EXZ2	-0.40083	0.291976	-0.04542	0.276517	0.139133	-0.33067	0.375211	-0.33466	-0.52332	0.451206
EXZ3	0.125472	-0.07772	0.038171	-0.13163	-0.10429	0.190421	-0.33485	0.124391	0.353857	-0.2141
EYZ2	0.397291	0.330945	0.552332	0.644622	-0.51807	-0.5877	0.192709	0.506274	0.052442	-0.30157
EYZ3	-0.21654	-0.0389	-0.46813	-0.37712	0.427918	0.374769	-0.27083	-0.37746	-0.25875	0.202235
EZZ1	-0.35789	0.018967	-0.44	-0.16717	0.463115	0.040233	0.213447	-0.2932	-0.55065	0.300906
EZZ2	0.452482	0.805281	0.173848	0.871287	-0.17312	-0.81299	-0.04984	0.417264	-0.57693	-0.342
EZZ3	-0.32537	-0.77394	-0.02667	-0.78397	0.013591	0.791152	-0.11022	-0.34164	0.737368	0.239382
EAZ0	-0.32607	-0.78125	-0.19493	-0.89195	0.172613	0.864964	0.055863	-0.41053	0.451853	0.324951
EAZ1	0.321629	0.762667	0.002478	0.8518	0.014581	-0.87595	-0.13077	0.439235	-0.57911	-0.40163
EAZ2	-0.35106	-0.78965	0.043229	-0.86684	-0.0587	0.890114	0.198994	-0.44891	0.650183	0.435801
EAZ3	0.349453	0.822619	-0.14552	0.827397	0.151919	-0.84156	-0.30319	0.391607	-0.75038	-0.42712
ECZ0	-0.69548	-0.19551	-0.24429	-0.40562	0.30035	0.359113	0.040715	-0.68362	-0.01859	0.605775
ECZ1	0.592807	0.203367	0.164181	0.380119	-0.21395	-0.36515	-0.0485	0.61979	0.006838	-0.56608
ECZ2	-0.46142	-0.21497	-0.16349	-0.33455	0.210456	0.318384	-0.00615	-0.51095	-0.00645	0.439346
ECZ3	0.292556	0.200635	0.133438	0.220495	-0.18195	-0.19694	0.033481	0.355319	0.001279	-0.27481
EXB0	-0.29272	-0.82911	0.002587	-0.79205	-0.00723	0.753052	0.350849	-0.20425	0.539235	0.297228
EXB1	0.997087	-0.12796	0.312035	0.096027	-0.39686	-0.0001	-0.07605	0.905063	0.133437	-0.78199

	EBB4	ECB1	ECB2	ECB3	EXC1	EXC2	EXC3	EYC1	EYC2	EYC3
EXX2	0.465676	-0.10425	0.808189	-0.15823	0.182555	-0.70912	-0.19808	0.40981	-0.24171	-0.38725
EXX3	0.720243	-0.03379	0.782929	-0.15162	0.34795	-0.6751	-0.36044	0.611786	-0.26611	-0.59381
EYX2	-0.63437	0.231669	-0.43228	-0.18353	-0.53872	0.110587	0.538177	-0.60719	0.35434	0.610982
EYX3	-0.78217	-0.2053	-0.70523	0.219833	-0.75483	0.758776	0.764764	-0.73564	0.63557	0.725357
EZX2	-0.44679	0.558755	-0.14619	-0.3835	-0.06626	-0.02389	0.088256	-0.07048	-0.33987	0.066887
EZX3	-0.13441	0.552917	0.389094	-0.66314	-0.25445	-0.44859	0.269131	0.114386	-0.10662	-0.1011
EAX0	-0.32808	-0.94457	-0.42944	0.904336	-0.14516	0.596576	0.132805	-0.59826	0.340234	0.594098
EAX1	-0.30959	-0.06748	0.084866	0.066313	0.010898	0.070833	0.005431	-0.20779	-0.25847	0.208642
EAX2	-0.44298	0.39571	0.132618	-0.35583	-0.03809	-0.326	0.047937	-0.21042	-0.29275	0.224428
EAX3	0.021651	0.21052	-0.26383	-0.13783	-0.03154	-0.02439	0.019882	0.034879	0.162333	-0.0373
EBX1	0.739362	-0.27622	0.48162	0.116602	0.330419	-0.40557	-0.35789	0.52453	-0.13621	-0.52095
EBX2	0.048374	-0.65124	-0.35928	0.763904	0.306767	0.427545	-0.32292	-0.19222	-0.00141	0.181551
EBX3	-0.26265	-0.59923	-0.41261	0.523163	-0.23264	0.533681	0.226377	-0.56137	0.67003	0.564572
ECX0	0.730992	0.165491	0.999058	-0.3164	0.392518	-0.86097	-0.4008	0.732302	-0.53421	-0.71427
ECX1	-0.15549	-0.67892	-0.33825	0.539036	-0.57152	0.486852	0.557018	-0.43178	0.668125	0.425959
ECX2	-0.89118	-0.13539	-0.68722	0.179484	-0.59995	0.684935	0.613209	-0.79546	0.499453	0.789306
ECX3	-0.51529	0.296852	-0.45619	-0.02912	0.300548	0.247492	-0.28267	-0.31295	-0.3124	0.306288
EXY0	-0.75243	-0.22219	-0.76845	0.22655	-0.51609	0.641003	0.520896	-0.74642	0.490843	0.733441
EXY2	0.153295	-0.36419	-0.3728	0.279453	-0.22502	0.407657	0.209031	-0.10381	0.53654	0.088187
EXY3	-0.23078	-0.55397	0.188331	0.546915	0.429523	-0.15776	-0.43938	-0.31922	-0.40209	0.337429
EYY0	0.209919	0.502774	0.112424	-0.4842	0.106467	-0.20983	-0.10497	0.318834	-0.07353	-0.32284
EYY1	0.378944	-0.17208	0.612892	0.00903	0.306921	-0.53278	-0.31443	0.28757	-0.25443	-0.26633
EYY2	0.231259	-0.39667	0.443242	0.353727	0.426939	-0.29877	-0.43226	0.197669	-0.49453	-0.1866
EYY3	-0.21812	0.22522	-0.51108	-0.10557	-0.36331	0.455578	0.368164	-0.14908	0.313268	0.127563
EZY2	0.548269	-0.44779	0.238381	0.299553	0.119142	-0.1246	-0.14053	0.147284	0.219397	-0.14411
EZY3	-0.08709	-0.27539	-0.38383	0.146907	-0.65829	0.583855	0.658786	-0.23986	0.829252	0.228005
EXZ1	-0.12424	-0.04789	-0.54943	0.339183	0.276379	0.484773	-0.27091	-0.13454	-0.1208	0.114495
EXZ2	-0.20613	-0.02633	0.276353	-0.23397	-0.34027	-0.25463	0.340133	-0.19467	0.297523	0.216651
EXZ3	0.417212	0.020778	-0.11363	0.132654	0.177722	0.158991	-0.18285	0.365112	-0.19488	-0.38861
EYZ2	0.274784	0.539843	0.626422	-0.49469	0.509817	-0.67965	-0.50254	0.561472	-0.7892	-0.55149
EYZ3	0.111096	-0.50372	-0.34904	0.403992	-0.32165	0.42813	0.306698	-0.23177	0.712183	0.225665
EZZ1	-0.32603	-0.39936	-0.16445	0.285152	-0.39558	0.141263	0.389762	-0.49468	0.410216	0.505694
EZZ2	0.784593	0.101151	0.872853	-0.20859	0.661494	-0.71031	-0.66645	0.796711	-0.59109	-0.78226
EZZ3	-0.56257	0.016721	-0.78229	0.105664	-0.50824	0.620885	0.511261	-0.58078	0.497863	0.562186
EAZ0	-0.76486	-0.14333	-0.88733	0.207828	-0.62621	0.757509	0.633609	-0.84452	0.767242	0.835062
EAZ1	0.684695	-0.03931	0.838656	-0.02909	0.665137	-0.7182	-0.67505	0.710304	-0.78813	-0.69932
EAZ2	-0.68403	0.094043	-0.85097	0.003842	-0.69183	0.729355	0.703339	-0.67176	0.739288	0.657805
EAZ3	0.734894	-0.21435	0.816104	0.068569	0.667748	-0.66768	-0.68299	0.635164	-0.56895	-0.62009
ECZ0	-0.49901	-0.2096	-0.3977	0.040381	-0.77272	0.440458	0.771411	-0.58831	0.842802	0.591196
ECZ1	0.450302	0.142504	0.369319	0.037631	0.709057	-0.38235	-0.70805	0.545555	-0.84891	-0.54984
ECZ2	-0.4844	-0.15013	-0.33458	-0.02043	-0.57461	0.321749	0.574119	-0.5728	0.756169	0.580582
ECZ3	0.518248	0.126102	0.236542	0.018924	0.374833	-0.19667	-0.37625	0.567436	-0.55032	-0.57996
EXB0	-0.7379	0.092632	-0.77929	-0.0256	-0.50683	0.587339	0.520983	-0.62729	0.430539	0.612185
EXB1	0.205879	0.262512	0.087391	-0.13501	0.808463	-0.25261	-0.80185	0.309427	-0.53866	-0.30736



	EZC1	EZC2	EZC3	EAC1	EAC2	EAC3	EBC1	EBC2	EBC3	ECC1
EXX2	0.867158	-0.24729	-0.81371	0.306985	0.104823	-0.29167	0.214736	0.708788	-0.20701	0.259483
EXX3	0.761528	-0.51012	-0.70756	0.524646	0.161895	-0.51555	0.114163	0.734922	-0.11722	0.536896
EYX2	-0.16745	0.592481	0.152598	-0.44504	0.108768	0.447182	0.307399	-0.16839	-0.28457	-0.65209
EYX3	-0.58855	0.142666	0.59823	-0.67426	0.099049	0.663088	0.183693	-0.78316	-0.17933	-0.68972
EZX2	-0.11961	0.662952	0.113247	0.113371	-0.48685	-0.11317	5.13E-05	-0.20383	0.006934	0.124584
EZX3	0.467008	0.344158	-0.43824	0.238204	-0.01248	-0.23136	0.510563	0.248934	-0.50883	-0.03744
EAX0	-0.33397	-0.50811	0.333208	-0.68768	0.001136	0.683807	-0.36914	-0.38608	0.376903	-0.224
EAX1	-0.05157	0.092642	0.073032	-0.19087	-0.48771	0.191022	-0.18154	-0.09822	0.165516	-0.2583
EAX2	0.203783	0.655345	-0.17806	-0.12846	-0.22395	0.140716	0.148693	0.206824	-0.13917	-0.06107
EAX3	-0.07189	0.227258	0.03237	0.086965	0.277557	-0.08496	0.087088	-0.03002	-0.06231	0.140892
EBX1	0.485921	-0.33518	-0.50143	0.384537	0.106025	-0.37837	-0.07544	0.463976	0.084377	0.166975
EBX2	-0.43506	-0.28931	0.417139	-0.33336	-0.13025	0.32949	-0.61491	-0.2178	0.616044	0.020489
EBX3	-0.4085	-0.57702	0.440381	-0.71319	0.641593	0.709401	0.002662	-0.26167	-0.01462	-0.31367
ECX0	0.884101	-0.09575	-0.8289	0.641249	-0.05915	-0.62972	0.208144	0.858979	-0.21534	0.533563
ECX1	-0.1332	-0.44997	0.12268	-0.4369	0.301471	0.427883	0.163528	-0.36454	-0.15049	-0.43751
ECX2	-0.60132	0.268057	0.610486	-0.75294	0.069352	0.746892	0.101424	-0.70913	-0.09901	-0.62084
ECX3	-0.58111	0.496919	0.557909	-0.26436	-0.39713	0.26557	-0.47303	-0.29268	0.465816	-0.04899
EXY0	-0.53364	0.156095	0.453595	-0.60692	-0.05993	0.601077	-0.06957	-0.72816	0.088032	-0.74607
EXY2	-0.29999	-0.31373	0.224746	-0.14457	0.312897	0.135373	-0.02161	-0.35159	0.026979	-0.52434
EXY3	0.183258	-0.07941	-0.13133	-0.46264	-0.27062	0.478848	-0.46969	0.330862	0.476096	0.239536
EYY0	-0.00123	0.421078	-0.03307	0.286709	0.138696	-0.28473	0.185846	0.096647	-0.19261	-0.12575
EYY1	0.642426	-0.52282	-0.57924	0.228187	0.061248	-0.21716	-0.01659	0.617002	0.016119	0.473788
EYY2	0.434695	-0.39138	-0.39429	0.147523	-0.36861	-0.13967	-0.3486	0.373589	0.352495	0.641791
EYY3	-0.53898	0.478953	0.475767	-0.09284	0.031547	0.080614	0.147122	-0.55295	-0.14725	-0.49466
EZY2	0.220206	-0.59063	-0.23435	0.031831	0.368893	-0.03239	0.0098	0.344774	-0.0167	-0.12344
EZY3	-0.3284	-0.43261	0.320119	-0.25374	0.560013	0.238157	0.396392	-0.53371	-0.4042	-0.46408
EXZ1	-0.70674	0.15559	0.657257	-0.13332	-0.26285	0.123345	-0.51391	-0.41084	0.505046	0.094453
EXZ2	0.452273	-0.18538	-0.39595	-0.21004	0.426422	0.220677	0.469169	0.230229	-0.46277	-0.1642
EXZ3	-0.24921	0.096946	0.185051	0.395175	-0.38724	-0.40801	-0.28217	-0.18133	0.278738	0.004036
EYZ2	0.478356	0.46376	-0.45144	0.553114	-0.45132	-0.54075	-0.07069	0.517945	0.067558	0.49965
EYZ3	-0.24125	-0.62392	0.222425	-0.28789	0.605179	0.278256	0.120056	-0.24071	-0.11924	-0.27625
EZZ1	0.092255	-0.37718	-0.05958	-0.42813	0.222922	0.429022	0.137737	-0.03496	-0.12328	-0.20928
EZZ2	0.664578	-0.35629	-0.61033	0.646981	-0.06341	-0.63665	-0.1005	0.736735	0.084762	0.721862
EZZ3	-0.6615	0.439829	0.58265	-0.46657	0.074645	0.45597	0.084302	-0.66217	-0.07409	-0.70618
EAZ0	-0.7244	0.064648	0.691563	-0.75421	0.382281	0.744281	0.14074	-0.70108	-0.13661	-0.68527
EAZ1	0.721682	-0.16109	-0.69361	0.628191	-0.46398	-0.61679	-0.27576	0.700187	0.275961	0.673055
EAZ2	-0.73439	0.246357	0.701192	-0.55967	0.363661	0.546436	0.279691	-0.74331	-0.27841	-0.64993
EAZ3	0.70558	-0.44475	-0.6709	0.478954	-0.14926	-0.46609	-0.25062	0.738005	0.246882	0.600317
ECZ0	-0.21961	-0.11861	0.246216	-0.60164	0.591329	0.597521	0.480911	-0.41169	-0.47607	-0.73039
ECZ1	0.20296	0.138483	-0.22207	0.569053	-0.6862	-0.56645	-0.49913	0.352663	0.496585	0.709858
ECZ2	-0.18671	-0.12069	0.206711	-0.62094	0.684052	0.622166	0.421615	-0.27317	-0.42022	-0.62683
ECZ3	0.123524	0.043612	-0.14809	0.637307	-0.596	-0.64383	-0.31311	0.139234	0.313069	0.463107
EXB0	-0.56801	0.268244	0.481721	-0.41677	-0.06891	0.408554	0.013637	-0.73954	0.000612	-0.60644
EXB1	-0.0622	-0.05187	0.026931	0.268972	-0.15836	-0.26006	-0.53236	0.235809	0.523241	0.579834



	ECC2	ECC3	ECC4	W	T
EXX2	-0.13138	-0.25775	0.153994	0.04764	0.3992
EXX3	-0.35154	-0.47279	-0.03901	0.053018	0.436413
EYX2	0.432968	0.612777	-0.27252	-0.30266	-0.42512
EYX3	0.294624	0.670016	-0.2508	-0.37985	-0.74611
EZX2	-0.39827	-0.31419	0.328219	-0.36167	-0.43923
EZX3	-0.55722	-0.16346	0.341938	-0.36615	-0.26668
EAX0	0.471934	0.341236	-0.47915	-0.15273	-0.28384
EAX1	-0.4657	-0.14286	0.583977	-0.36625	-0.31893
EAX2	-0.03868	-0.08122	0.322957	-0.22759	-0.1265
EAX3	0.525217	0.193419	-0.51819	0.243017	0.173599
EBX1	0.239475	-0.07051	0.026829	0.488001	0.677353
EBX2	0.613386	0.13537	-0.26175	0.319615	0.262499
EBX3	0.599344	0.619528	-0.67176	0.054299	-0.02989
ECX0	-0.46035	-0.64091	0.520127	0.216631	0.551018
ECX1	0.335305	0.538411	-0.54326	-0.17978	-0.42458
ECX2	0.352168	0.617578	-0.16522	-0.32434	-0.6331
ECX3	0.109478	-0.03245	0.188774	-0.04263	-0.049
EXY0	0.287859	0.676015	-0.29462	-0.36936	-0.66079
EXY2	0.460988	0.623805	-0.31688	0.296609	0.095458
EXY3	0.358178	-0.19475	0.017296	-0.03942	0.221791
EYY0	0.135764	0.11841	0.405487	0.505206	0.474861
EYY1	-0.33938	-0.43062	-0.1638	-0.2195	0.180512
EYY2	-0.35066	-0.66583	0.030614	-0.21066	0.041751
EYY3	0.294441	0.450379	0.182708	0.28218	-0.11997
EZY2	0.189377	0.189549	-0.22236	0.30112	0.408025
EZY3	0.111216	0.589813	-0.40775	-0.04394	-0.37528
EXZ1	0.233915	-0.04811	-0.01609	0.257504	0.054486
EXZ2	-0.03173	0.23944	-0.23024	-0.33391	-0.17004
EXZ3	-0.09756	-0.16698	0.317129	0.325013	0.150018
EYZ2	-0.4188	-0.67849	0.731781	0.140033	0.410132
EYZ3	0.424892	0.548653	-0.74123	0.098022	-0.05896
EZZ1	0.076052	0.283449	-0.66584	-0.57258	-0.52706
EZZ2	-0.44846	-0.72015	0.371055	0.275226	0.684908
EZZ3	0.478265	0.689896	-0.15435	0.027134	-0.39396
EAZ0	0.483628	0.816662	-0.60885	-0.25977	-0.59746
EAZ1	-0.45538	-0.81712	0.505561	0.142731	0.515915
EAZ2	0.377754	0.749957	-0.4325	-0.16798	-0.58406
EAZ3	-0.24621	-0.60423	0.251817	0.237787	0.661698
ECZ0	0.450849	0.863078	-0.46647	-0.21853	-0.41667
ECZ1	-0.46412	-0.87902	0.485354	0.145298	0.309718
ECZ2	0.514565	0.8273	-0.46687	-0.0965	-0.2048
ECZ3	-0.5036	-0.66643	0.357867	0.055302	0.084659
EXB0	0.033984	0.519836	-0.26582	-0.43577	-0.74265
EXB1	-0.11459	-0.46102	0.040727	0.246192	0.523654

	EXX2	EXX3	EYX2	EYX3	EZX2	EZX3	EAX0	EAX1	EAX2
EXB2	0.730618	0.787955	-0.49522	-0.71681	-0.45839	-0.01949	-0.19517	-0.11554	-0.10376
EXB3	0.041639	-0.30362	0.251787	0.705287	-0.20241	-0.02896	0.681488	0.433949	-0.04509
EYB1	0.043938	-0.07471	-0.03852	0.208979	-0.05241	-0.27284	0.740429	0.359315	0.040046
EYB2	0.042861	-0.33072	0.409634	0.461486	0.714572	0.719675	-0.34913	0.383142	0.595995
EYB3	0.010015	0.183997	-0.05736	-0.38691	0.086597	0.269206	-0.84394	-0.33633	0.002194
EZB1	-0.60497	-0.7187	0.530459	0.762787	0.450877	0.122264	0.236891	0.144603	0.132447
EZB2	-0.11559	0.206521	-0.29581	-0.68285	0.086764	-0.28528	-0.33333	-0.31091	-0.02618
EZB3	0.816667	0.821381	-0.50134	-0.53643	-0.39812	0.156449	-0.10769	-0.01056	-0.1273
EAB0	-0.06527	0.027666	0.176604	-0.28897	0.485029	0.503693	-0.96236	-0.14066	0.350883
EAB1	0.815159	0.766344	-0.4236	-0.70107	-0.14074	0.377801	-0.40165	0.112394	0.165089
EAB2	0.147013	-0.0107	-0.13853	0.318985	-0.46386	-0.41127	0.944987	0.168232	-0.28338
EAB3	-0.85033	-0.74068	0.409711	0.602241	0.139171	-0.35706	0.210222	-0.21065	-0.18905
EBB0	-0.01081	0.105127	0.19369	0.069487	0.62472	0.776961	-0.58079	-0.00603	0.319599
EBB1	-0.00067	0.198223	-0.17138	-0.61373	0.216386	-0.17764	-0.31985	-0.18607	0.106034
EBB2	-0.79865	-0.8469	0.310793	0.385491	0.503112	-0.08039	-0.28035	0.053746	0.186737
EBB3	0.055436	-0.07451	0.285016	0.493038	0.058003	0.496655	-0.07823	0.071004	0.049753
EBB4	0.465676	0.720243	-0.63437	-0.78217	-0.44679	-0.13441	-0.32808	-0.30959	-0.44298
ECB1	-0.10425	-0.03379	0.231669	-0.2053	0.558755	0.552917	-0.94457	-0.06748	0.39571
ECB2	0.808189	0.782929	-0.43228	-0.70523	-0.14619	0.389094	-0.42944	0.084866	0.132618
ECB3	-0.15823	-0.15162	-0.18353	0.219833	-0.3835	-0.66314	0.904336	0.066313	-0.35583
EXC1	0.182555	0.34795	-0.53872	-0.75483	-0.06626	-0.25445	-0.14516	0.010898	-0.03809
EXC2	-0.70912	-0.6751	0.110587	0.758776	-0.02389	-0.44859	0.596576	0.070833	-0.326
EXC3	-0.19808	-0.36044	0.538177	0.764764	0.088256	0.269131	0.132805	0.005431	0.047937
EYC1	0.40981	0.611786	-0.60719	-0.73564	-0.07048	0.114386	-0.59826	-0.20779	-0.21042
EYC2	-0.24171	-0.26611	0.35434	0.63557	-0.33987	-0.10662	0.340234	-0.25847	-0.29275
EYC3	-0.38725	-0.59381	0.610982	0.725357	0.066887	-0.1011	0.594098	0.208642	0.224428
EZC1	0.867158	0.761528	-0.16745	-0.58855	-0.11961	0.467008	-0.33397	-0.05157	0.203783
EZC2	-0.24729	-0.51012	0.592481	0.142666	0.662952	0.344158	-0.50811	0.092642	0.655345
EZC3	-0.81371	-0.70756	0.152598	0.59823	0.113247	-0.43824	0.333208	0.073032	-0.17806
EAC1	0.306985	0.524646	-0.44504	-0.67426	0.113371	0.238204	-0.68768	-0.19087	-0.12846
EAC2	0.104823	0.161895	0.108768	0.099049	-0.48685	-0.01248	0.001136	-0.48771	-0.22395
EAC3	-0.29167	-0.51555	0.447182	0.663088	-0.11317	-0.23136	0.683807	0.191022	0.140716
EBC1	0.214736	0.114163	0.307399	0.183693	5.13E-05	0.510563	-0.36914	-0.18154	0.148693
EBC2	0.708788	0.734922	-0.16839	-0.78316	-0.20383	0.248934	-0.38608	-0.09822	0.206824
EBC3	-0.20701	-0.11722	-0.28457	-0.17933	0.006934	-0.50883	0.376903	0.165516	-0.13917
ECC1	0.259483	0.536896	-0.65209	-0.68972	0.124584	-0.03744	-0.224	-0.2583	-0.06107
ECC2	-0.13138	-0.35154	0.432968	0.294624	-0.39827	-0.55722	0.471934	-0.4657	-0.03868
ECC3	-0.25775	-0.47279	0.612777	0.670016	-0.31419	-0.16346	0.341236	-0.14286	-0.08122
ECC4	0.153994	-0.03901	-0.27252	-0.2508	0.328219	0.341938	-0.47915	0.583977	0.322957
W	0.04764	0.053018	-0.30266	-0.37985	-0.36167	-0.36615	-0.15273	-0.36625	-0.22759
T	0.3992	0.436413	-0.42512	-0.74611	-0.43923	-0.26668	-0.28384	-0.31893	-0.1265

	EAX3	EBX1	EBX2	EBX3	ECX0	ECX1	ECX2	ECX3	EXY0	EXY2
EXB2	-0.13581	0.668861	0.05423	-0.08197	0.879879	-0.22229	-0.71516	-0.43457	-0.86478	-0.15763
EXB3	-0.443	-0.04187	0.14135	0.301407	-0.23307	0.764817	0.56464	-0.34428	0.56852	0.325709
EYB1	-0.31766	-0.06915	0.452946	0.218079	-0.11542	0.338724	0.221157	0.055851	0.194928	-0.13549
EYB2	-0.14775	-0.42739	-0.65643	-0.6145	0.034389	-0.09523	0.452068	0.029437	0.303921	-0.38524
EYB3	0.289043	0.102605	-0.43408	-0.31192	0.26126	-0.52052	-0.36876	0.019916	-0.3996	-0.0095
EZB1	0.099683	-0.64258	-0.14574	0.061116	-0.82522	0.326555	0.733219	0.294422	0.914231	0.150156
EZB2	0.360584	0.116154	0.255301	-0.15966	0.081948	-0.68789	-0.5146	0.522645	-0.44808	-0.22853
EZB3	-0.25857	0.606503	-0.12428	-0.13944	0.886753	-0.00274	-0.61474	-0.66404	-0.71796	-0.1569
EAB0	0.252457	-0.19233	-0.61157	-0.57391	0.220278	-0.69874	-0.21509	0.263428	-0.3054	-0.33646
EAB1	-0.27944	0.476194	-0.33151	-0.40377	0.999262	-0.34974	-0.66718	-0.4243	-0.76492	-0.39556
EAB2	-0.27533	0.195739	0.530081	0.540495	-0.16336	0.720858	0.245174	-0.3272	0.314979	0.302474
EAB3	0.361612	-0.45024	0.266048	0.358632	-0.9616	0.225633	0.574549	0.450729	0.666906	0.418548
EBB0	0.070292	-0.54414	-0.89786	-0.44004	0.094449	-0.19363	0.036579	-0.10841	0.007581	-0.50751
EBB1	0.333054	0.155379	0.210978	-0.36602	0.091034	-0.67932	-0.4669	0.535268	-0.31347	-0.26293
EBB2	0.193488	-0.50339	0.036804	-0.17206	-0.6035	-0.32207	0.482839	0.711437	0.382365	0.069547
EBB3	-0.15551	-0.25694	-0.58396	0.057277	-0.00253	0.450562	0.341701	-0.52194	0.24483	0.067288
EBB4	0.021651	0.739362	0.048374	-0.26265	0.730992	-0.15549	-0.89118	-0.51529	-0.75243	0.153295
ECB1	0.21052	-0.27622	-0.65124	-0.59923	0.165491	-0.67892	-0.13539	0.296852	-0.22219	-0.36419
ECB2	-0.26383	0.48162	-0.35928	-0.41261	0.999058	-0.33825	-0.68722	-0.45619	-0.76845	-0.3728
ECB3	-0.13783	0.116602	0.763904	0.523163	-0.3164	0.539036	0.179484	-0.02912	0.22655	0.279453
EXC1	-0.03154	0.330419	0.306767	-0.23264	0.392518	-0.57152	-0.59995	0.300548	-0.51609	-0.22502
EXC2	-0.02439	-0.40557	0.427545	0.533681	-0.86097	0.486852	0.684935	0.247492	0.641003	0.407657
EXC3	0.019882	-0.35789	-0.32292	0.226377	-0.4008	0.557018	0.613209	-0.28267	0.520896	0.209031
EYC1	0.034879	0.52453	-0.19222	-0.56137	0.732302	-0.43178	-0.79546	-0.31295	-0.74642	-0.10381
EYC2	0.162333	-0.13621	-0.00141	0.67003	-0.53421	0.668125	0.499453	-0.3124	0.490843	0.53654
EYC3	-0.0373	-0.52095	0.181551	0.564572	-0.71427	0.425959	0.789306	0.306288	0.733441	0.088187
EZC1	-0.07189	0.485921	-0.43506	-0.4085	0.884101	-0.1332	-0.60132	-0.58111	-0.53364	-0.29999
EZC2	0.227258	-0.33518	-0.28931	-0.57702	-0.09575	-0.44997	0.268057	0.496919	0.156095	-0.31373
EZC3	0.03237	-0.50143	0.417139	0.440381	-0.8289	0.12268	0.610486	0.557909	0.453595	0.224746
EAC1	0.086965	0.384537	-0.33336	-0.71319	0.641249	-0.4369	-0.75294	-0.26436	-0.60692	-0.14457
EAC2	0.277557	0.106025	-0.13025	0.641593	-0.05915	0.301471	0.069352	-0.39713	-0.05993	0.312897
EAC3	-0.08496	-0.37837	0.32949	0.709401	-0.62972	0.427883	0.746892	0.26557	0.601077	0.135373
EBC1	0.087088	-0.07544	-0.61491	0.002662	0.208144	0.163528	0.101424	-0.47303	-0.06957	-0.02161
EBC2	-0.03002	0.463976	-0.2178	-0.26167	0.858979	-0.36454	-0.70913	-0.29268	-0.72816	-0.35159
EBC3	-0.06231	0.084377	0.616044	-0.01462	-0.21534	-0.15049	-0.09901	0.465816	0.088032	0.026979
ECC1	0.140892	0.166975	0.020489	-0.31367	0.533563	-0.43751	-0.62084	-0.04899	-0.74607	-0.52434
ECC2	0.525217	0.239475	0.613386	0.599344	-0.46035	0.335305	0.352168	0.109478	0.287859	0.460988
ECC3	0.193419	-0.07051	0.13537	0.619528	-0.64091	0.538411	0.617578	-0.03245	0.676015	0.623805
ECC4	-0.51819	0.026829	-0.26175	-0.67176	0.520127	-0.54326	-0.16522	0.188774	-0.29462	-0.31688
W	0.243017	0.488001	0.319615	0.054299	0.216631	-0.17978	-0.32434	-0.04263	-0.36936	0.296609
T	0.173599	0.677353	0.262499	-0.02989	0.551018	-0.42458	-0.6331	-0.049	-0.66079	0.095458

	EXY3	EYY0	EYY1	EYY2	EYY3	EZY2	EZY3	EXZ1	EXZ2	EXZ3
EXB2	0.31992	0.101829	0.584905	0.400122	-0.48841	0.472779	-0.31771	-0.33348	0.111776	-0.00777
EXB3	0.177348	-0.33663	-0.17235	-0.00861	0.171771	0.147771	0.486222	-0.24936	0.229718	-0.02902
EYB1	0.747674	-0.58953	0.196221	0.552028	-0.32881	0.019475	-0.16073	0.08502	-0.01077	-0.09873
EYB2	-0.20712	0.100174	-0.27902	-0.14796	0.282906	-0.71481	-0.04417	-0.36507	0.270479	-0.14866
EYB3	-0.65709	0.609297	-0.10068	-0.44973	0.224804	-0.03035	-0.02811	-0.04088	-0.0621	0.130402
EZB1	-0.30543	-0.18089	-0.49743	-0.33626	0.414349	-0.46689	0.364901	0.162738	0.035394	-0.08089
EZB2	0.167622	0.186589	0.169586	0.160071	-0.23773	-0.05303	-0.6191	0.460856	-0.40083	0.125472
EZB3	0.211567	-0.0696	0.651986	0.491153	-0.53222	0.408535	-0.12137	-0.55545	0.291976	-0.07772
EAB0	-0.54646	0.54302	-0.14505	-0.39227	0.209277	-0.38001	-0.2866	-0.03785	-0.04542	0.038171
EAB1	0.257839	0.105029	0.612644	0.463875	-0.52294	0.226545	-0.42273	-0.53906	0.276517	-0.13163
EAB2	0.564297	-0.53817	0.163276	0.399677	-0.21965	0.351666	0.292864	-0.06875	0.139133	-0.10429
EAB3	-0.42099	0.051242	-0.65972	-0.59319	0.597946	-0.20735	0.42255	0.576413	-0.33067	0.190421
EBB0	-0.54621	-0.09211	0.150863	-0.00842	-0.11384	-0.51177	0.07484	-0.37484	0.375211	-0.33485
EBB1	0.246345	0.13076	0.202801	0.229438	-0.27886	-0.18169	-0.74903	0.327649	-0.33466	0.124391
EBB2	-0.39348	0.471622	-0.87311	-0.65646	0.808072	-0.52272	0.014744	0.643141	-0.52332	0.353857
EBB3	-0.51238	-0.05478	-0.10844	-0.27612	0.214696	-0.03797	0.632966	-0.48581	0.451206	-0.2141
EBB4	-0.23078	0.209919	0.378944	0.231259	-0.21812	0.548269	-0.08709	-0.12424	-0.20613	0.417212
ECB1	-0.55397	0.502774	-0.17208	-0.39667	0.22522	-0.44779	-0.27539	-0.04789	-0.02633	0.020778
ECB2	0.188331	0.112424	0.612892	0.443242	-0.51108	0.238381	-0.38383	-0.54943	0.276353	-0.11363
ECB3	0.546915	-0.4842	0.00903	0.353727	-0.10557	0.299553	0.146907	0.339183	-0.23397	0.132654
EXC1	0.429523	0.106467	0.306921	0.426939	-0.36331	0.119142	-0.65829	0.276379	-0.34027	0.177722
EXC2	-0.15776	-0.20983	-0.53278	-0.29877	0.455578	-0.1246	0.583855	0.484773	-0.25463	0.158991
EXC3	-0.43938	-0.10497	-0.31443	-0.43226	0.368164	-0.14053	0.658786	-0.27091	0.340133	-0.18285
EYC1	-0.31922	0.318834	0.28757	0.197669	-0.14908	0.147284	-0.23986	-0.13454	-0.19467	0.365112
EYC2	-0.40209	-0.07353	-0.25443	-0.49453	0.313268	0.219397	0.829252	-0.1208	0.297523	-0.19488
EYC3	0.337429	-0.32284	-0.26633	-0.1866	0.127563	-0.14411	0.228005	0.114495	0.216651	-0.38861
EZC1	0.183258	-0.00123	0.642426	0.434695	-0.53898	0.220206	-0.3284	-0.70674	0.452273	-0.24921
EZC2	-0.07941	0.421078	-0.52282	-0.39138	0.478953	-0.59063	-0.43261	0.15559	-0.18538	0.096946
EZC3	-0.13133	-0.03307	-0.57924	-0.39429	0.475767	-0.23435	0.320119	0.657257	-0.39595	0.185051
EAC1	-0.46264	0.286709	0.228187	0.147523	-0.09284	0.031831	-0.25374	-0.13332	-0.21004	0.395175
EAC2	-0.27062	0.138696	0.061248	-0.36861	0.031547	0.368893	0.560013	-0.26285	0.426422	-0.38724
EAC3	0.478848	-0.28473	-0.21716	-0.13967	0.080614	-0.03239	0.238157	0.123345	0.220677	-0.40801
EBC1	-0.46969	0.185846	-0.01659	-0.3486	0.147122	0.0098	0.396392	-0.51391	0.469169	-0.28217
EBC2	0.330862	0.096647	0.617002	0.373589	-0.55295	0.344774	-0.53371	-0.41084	0.230229	-0.18133
EBC3	0.476096	-0.19261	0.016119	0.352495	-0.14725	-0.0167	-0.4042	0.505046	-0.46277	0.278738
ECC1	0.239536	-0.12575	0.473788	0.641791	-0.49466	-0.12344	-0.46408	0.094453	-0.1642	0.004036
ECC2	0.358178	0.135764	-0.33938	-0.35066	0.294441	0.189377	0.111216	0.233915	-0.03173	-0.09756
ECC3	-0.19475	0.11841	-0.43062	-0.66583	0.450379	0.189549	0.589813	-0.04811	0.23944	-0.16698
ECC4	0.017296	0.405487	-0.1638	0.030614	0.182708	-0.22236	-0.40775	-0.01609	-0.23024	0.317129
W	-0.03942	0.505206	-0.2195	-0.21066	0.28218	0.30112	-0.04394	0.257504	-0.33391	0.325013
T	0.221791	0.474861	0.180512	0.041751	-0.11997	0.408025	-0.37528	0.054486	-0.17004	0.150018

	EY22	EY23	EZZ1	EZZ2	EZZ3	EAZ0	EAZ1	EAZ2	EAZ3	ECZ0
EXB2	0.422619	-0.07397	-0.16573	0.876625	-0.75435	-0.80671	0.770755	-0.82124	0.870155	-0.35175
EXB3	-0.48777	0.255225	0.402712	-0.50395	0.357587	0.35409	-0.25361	0.265518	-0.25547	0.605425
EYB1	-0.16944	-0.00347	0.388582	-0.12033	-0.08561	0.031246	0.176372	-0.17944	0.145866	-0.03865
EYB2	0.369735	-0.60044	-0.063	-0.30664	0.215454	0.000586	-0.08199	0.188747	-0.39506	0.211497
EYB3	0.34305	-0.126	-0.47626	0.303104	-0.07513	-0.21077	-0.00229	0.007231	0.002977	-0.15456
EZB1	-0.44437	0.086951	0.287817	-0.87909	0.708956	0.789397	-0.73619	0.786891	-0.83691	0.441244
EZB2	0.397291	-0.21654	-0.35789	0.452482	-0.32537	-0.32607	0.321629	-0.35106	0.349453	-0.69548
EZB3	0.330945	-0.0389	0.018967	0.805281	-0.77394	-0.78125	0.762667	-0.78965	0.822619	-0.19551
EAB0	0.552332	-0.46813	-0.44	0.173848	-0.02667	-0.19493	0.002478	0.043229	-0.14552	-0.24429
EAB1	0.644622	-0.37712	-0.16717	0.871287	-0.78397	-0.89195	0.8518	-0.86684	0.827397	-0.40562
EAB2	-0.51807	0.427918	0.463115	-0.17312	0.013591	0.172613	0.014581	-0.0587	0.151919	0.30035
EAB3	-0.5877	0.374769	0.040233	-0.81299	0.791152	0.864964	-0.87595	0.890114	-0.84156	0.359113
EBB0	0.192709	-0.27083	0.213447	-0.04984	-0.11022	0.055863	-0.13077	0.198994	-0.30319	0.040715
EBB1	0.506274	-0.37746	-0.2932	0.417264	-0.34164	-0.41053	0.439235	-0.44891	0.391607	-0.68362
EBB2	0.052442	-0.25875	-0.55065	-0.57693	0.737368	0.451853	-0.57911	0.650183	-0.75038	-0.01859
EBB3	-0.30157	0.202235	0.300906	-0.342	0.239382	0.324951	-0.40163	0.435801	-0.42712	0.605775
EBB4	0.274784	0.111096	-0.32603	0.784593	-0.56257	-0.76486	0.684695	-0.68403	0.734894	-0.49901
ECB1	0.539843	-0.50372	-0.39936	0.101151	0.016721	-0.14333	-0.03931	0.094043	-0.21435	-0.2096
ECB2	0.626422	-0.34904	-0.16445	0.872853	-0.78229	-0.88733	0.838656	-0.85097	0.816104	-0.3977
ECB3	-0.49469	0.403992	0.285152	-0.20859	0.105664	0.207828	-0.02909	0.003842	0.068569	0.040381
EXC1	0.509817	-0.32165	-0.39558	0.661494	-0.50824	-0.62621	0.665137	-0.69183	0.667748	-0.77272
EXC2	-0.67965	0.42813	0.141263	-0.71031	0.620885	0.757509	-0.7182	0.729355	-0.66768	0.440458
EXC3	-0.50254	0.306698	0.389762	-0.66645	0.511261	0.633609	-0.67505	0.703339	-0.68299	0.771411
EYC1	0.561472	-0.23177	-0.49468	0.796711	-0.58078	-0.84452	0.710304	-0.67176	0.635164	-0.58831
EYC2	-0.7892	0.712183	0.410216	-0.59109	0.497863	0.767242	-0.78813	0.739288	-0.56895	0.842802
EYC3	-0.55149	0.225665	0.505694	-0.78226	0.562186	0.835062	-0.69932	0.657805	-0.62009	0.591196
EZC1	0.478356	-0.24125	0.092255	0.664578	-0.6615	-0.7244	0.721682	-0.73439	0.70558	-0.21961
EZC2	0.46376	-0.62392	-0.37718	-0.35629	0.439829	0.064648	-0.16109	0.246357	-0.44475	-0.11861
EZC3	-0.45144	0.222425	-0.05958	-0.61033	0.58265	0.691563	-0.69361	0.701192	-0.6709	0.246216
EAC1	0.553114	-0.28789	-0.42813	0.646981	-0.46657	-0.75421	0.628191	-0.55967	0.478954	-0.60164
EAC2	-0.45132	0.605179	0.222922	-0.06341	0.074645	0.382281	-0.46398	0.363661	-0.14926	0.591329
EAC3	-0.54075	0.278256	0.429022	-0.63665	0.45597	0.744281	-0.61679	0.546436	-0.46609	0.597521
EBC1	-0.07069	0.120056	0.137737	-0.1005	0.084302	0.14074	-0.27576	0.279691	-0.25062	0.480911
EBC2	0.517945	-0.24071	-0.03496	0.736735	-0.66217	-0.70108	0.700187	-0.74331	0.738005	-0.41169
EBC3	0.067558	-0.11924	-0.12328	0.084762	-0.07409	-0.13661	0.275961	-0.27841	0.246882	-0.47607
ECC1	0.49965	-0.27625	-0.20928	0.721862	-0.70618	-0.68527	0.673055	-0.64993	0.600317	-0.73039
ECC2	-0.4188	0.424892	0.076052	-0.44846	0.478265	0.483628	-0.45538	0.377754	-0.24621	0.450849
ECC3	-0.67849	0.548653	0.283449	-0.72015	0.689896	0.816662	-0.81712	0.749957	-0.60423	0.863078
ECC4	0.731781	-0.74123	-0.66584	0.371055	-0.15435	-0.60885	0.505561	-0.4325	0.251817	-0.46647
W	0.140033	0.098022	-0.57258	0.275226	0.027134	-0.25977	0.142731	-0.16798	0.237787	-0.21853
T	0.410132	-0.05896	-0.52706	0.684908	-0.39396	-0.59746	0.515915	-0.58406	0.661698	-0.41667

	ECZ1	ECZ2	ECZ3	EXB0	EXB1	EXB2	EXB3	EYB1	EYB2	EYB3
EXB2	0.318639	-0.27513	0.200029	-0.97184	0.13965	1	-0.19224	0.012808	-0.25274	0.147779
EXB3	-0.47722	0.378979	-0.24647	0.2889	-0.8942	-0.19224	1	0.538158	0.311944	-0.6717
EYB1	0.148958	-0.11203	0.038915	-0.02366	-0.17558	0.012808	0.538158	1	-0.09195	-0.9686
EYB2	-0.12147	0.050214	-0.03109	0.386206	-0.51783	-0.25274	0.311944	-0.09195	1	0.056948
EYB3	0.038822	-0.05247	0.080204	-0.15672	0.322844	0.147779	-0.6717	-0.9686	0.056948	1
EZB1	-0.39348	0.333662	-0.24029	0.995929	-0.26826	-0.98037	0.300505	0.016207	0.328509	-0.19802
EZB2	0.592807	-0.46142	0.292556	-0.29272	0.997087	0.147648	-0.86049	-0.11008	-0.53349	0.261871
EZB3	0.203367	-0.21497	0.200635	-0.82911	-0.12796	0.933069	0.023057	0.035202	-0.05088	0.076991
EAB0	0.164181	-0.16349	0.133438	0.002587	0.312035	-0.00222	-0.6504	-0.8317	0.359852	0.891286
EAB1	0.380119	-0.33455	0.220495	-0.79205	0.096027	0.880304	-0.23012	-0.08527	0.031738	0.234535
EAB2	-0.21395	0.210456	-0.18195	-0.00723	-0.39686	0.030566	0.707113	0.831223	-0.27355	-0.89746
EAB3	-0.36515	0.318384	-0.19694	0.753052	-0.0001	-0.85496	0.058282	-0.14939	-0.05493	0.007734
EBB0	-0.0485	-0.00615	0.033481	0.350849	-0.07605	-0.24613	-0.2886	-0.56902	0.526771	0.535022
EBB1	0.61979	-0.51095	0.355319	-0.20425	0.905063	0.109595	-0.71961	0.021269	-0.33672	0.126252
EBB2	0.006838	-0.00645	0.001279	0.539235	0.133437	-0.65441	-0.16167	-0.27939	0.317872	0.248041
EBB3	-0.56608	0.439346	-0.27481	0.297228	-0.78199	-0.16001	0.439105	-0.37551	0.52745	0.233379
EBB4	0.450302	-0.4844	0.518248	-0.7379	0.205879	0.778494	-0.34065	-0.35824	-0.27347	0.46011
ECB1	0.142504	-0.15013	0.126102	0.092632	0.262512	-0.0842	-0.5962	-0.80081	0.432009	0.849737
ECB2	0.369319	-0.33458	0.236542	-0.77929	0.087391	0.87342	-0.24648	-0.1461	0.036823	0.288482
ECB3	0.037631	-0.02043	0.018924	-0.0256	-0.13501	-0.03681	0.515624	0.869553	-0.39637	-0.88527
EXC1	0.709057	-0.57461	0.374833	-0.50683	0.808463	0.411343	-0.59373	0.178753	-0.5008	-0.00343
EXC2	-0.38235	0.321749	-0.19667	0.587339	-0.25261	-0.66681	0.41623	0.26565	-0.04831	-0.38955
EXC3	-0.70805	0.574119	-0.37625	0.520983	-0.80185	-0.4272	0.583141	-0.18831	0.511437	0.013744
EYC1	0.545555	-0.5728	0.567436	-0.62729	0.309427	0.676527	-0.50479	-0.47462	0.016435	0.607527
EYC2	-0.84891	0.756169	-0.55032	0.430539	-0.53866	-0.40757	0.40857	-0.25491	-0.05113	0.053652
EYC3	-0.54984	0.580582	-0.57996	0.612185	-0.30736	-0.65991	0.500921	0.478131	-0.01996	-0.60859
EZC1	0.20296	-0.18671	0.123524	-0.56801	-0.0622	0.713223	-0.10326	-0.11391	0.111275	0.186794
EZC2	0.138483	-0.12069	0.043612	0.268244	-0.05187	-0.27591	-0.05095	-0.13729	0.662813	0.163368
EZC3	-0.22207	0.206711	-0.14809	0.481721	0.026931	-0.62551	0.125984	0.151333	-0.08455	-0.20479
EAC1	0.569053	-0.62094	0.637307	-0.41677	0.268972	0.488466	-0.50512	-0.55506	0.160347	0.659913
EAC2	-0.6862	0.684052	-0.596	-0.06891	-0.15836	0.064787	-0.06967	-0.50176	-0.31434	0.396481
EAC3	-0.56645	0.622166	-0.64383	0.408554	-0.26006	-0.47975	0.498342	0.559357	-0.1643	-0.66226
EBC1	-0.49913	0.421615	-0.31311	0.013637	-0.53236	0.09498	0.123947	-0.58472	0.417605	0.502095
EBC2	0.352663	-0.27317	0.139234	-0.73954	0.235809	0.782326	-0.33192	-0.08959	-0.17732	0.226437
EBC3	0.496585	-0.42022	0.313069	0.000612	0.523241	-0.10459	-0.11115	0.59636	-0.40608	-0.51775
ECC1	0.709858	-0.62683	0.463107	-0.60644	0.579834	0.574453	-0.56425	0.124038	-0.18696	0.066153
ECC2	-0.46412	0.514565	-0.5036	0.033984	-0.11459	-0.11826	0.302811	0.316756	-0.27434	-0.38892
ECC3	-0.87902	0.8273	-0.66643	0.519836	-0.46102	-0.52448	0.449368	-0.14993	-0.05521	-0.05472
ECC4	0.485354	-0.46687	0.357867	-0.26582	0.040727	0.28676	-0.10418	-0.08422	0.433062	0.20719
W	0.145298	-0.0965	0.055302	-0.43577	0.246192	0.361939	-0.26485	-0.22607	-0.28414	0.276819
T	0.309718	-0.2048	0.084659	-0.74265	0.523654	0.675207	-0.50257	-0.1965	-0.47993	0.324227

	EZB1	EZB2	EZB3	EAB0	EAB1	EAB2	EAB3	EBB0	EBB1	EBB2
EXB2	-0.98037	0.147648	0.933069	-0.00222	0.880304	0.030566	-0.85496	-0.24613	0.109595	-0.65441
EXB3	0.300505	-0.86049	0.023057	-0.6504	-0.23012	0.707113	0.058282	-0.2886	-0.71961	-0.16167
EYB1	0.016207	-0.11008	0.035202	-0.8317	-0.08527	0.831223	-0.14939	-0.56902	0.021269	-0.27939
EYB2	0.328509	-0.53349	-0.05088	0.359852	0.031738	-0.27355	-0.05493	0.526771	-0.33672	0.317872
EYB3	-0.19802	0.261871	0.076991	0.891286	0.234535	-0.89746	0.007734	0.535022	0.126252	0.248041
EZB1	1	-0.27664	-0.85473	-0.04457	-0.8261	0.035421	0.780374	0.300278	-0.1972	0.52853
EZB2	-0.27664	1	-0.12229	0.255114	0.094023	-0.34173	-0.01276	-0.13408	0.920141	0.117385
EZB3	-0.85473	-0.12229	1	-0.06631	0.882842	0.121976	-0.88953	-0.09719	-0.09827	-0.75753
EAB0	-0.04457	0.255114	-0.06631	1	0.202829	-0.99183	0.006178	0.696233	0.20416	0.401775
EAB1	-0.8261	0.094023	0.882842	0.202829	1	-0.14607	-0.9689	0.077765	0.10846	-0.60766
EAB2	0.035421	-0.34173	0.121976	-0.99183	-0.14607	1	-0.06973	-0.65287	-0.27451	-0.44272
EAB3	0.780374	-0.01276	-0.88953	0.006178	-0.9689	-0.06973	1	0.027561	-0.07426	0.688809
EBB0	0.300278	-0.13408	-0.09719	0.696233	0.077765	-0.65287	0.027561	1	-0.15158	0.125401
EBB1	-0.1972	0.920141	-0.09827	0.20416	0.10846	-0.27451	-0.07426	-0.15158	1	0.057423
EBB2	0.52853	0.117385	-0.75753	0.401775	-0.60766	-0.44272	0.688809	0.125401	0.057423	1
EBB3	0.265509	-0.82525	0.078342	0.220164	-0.02734	-0.13943	0.068356	0.581379	-0.86817	-0.01421
EBB4	-0.77303	0.18913	0.745219	0.227522	0.715232	-0.22671	-0.60553	-0.0617	0.133112	-0.43972
ECB1	0.043981	0.207296	-0.12614	0.993393	0.14881	-0.98242	0.047892	0.734533	0.179984	0.439845
ECB2	-0.81666	0.080924	0.885745	0.242628	0.997091	-0.18597	-0.95317	0.122162	0.090063	-0.60542
ECB3	0.02693	-0.07643	-0.06004	-0.93602	-0.29542	0.902019	0.100606	-0.76632	-0.05791	-0.19085
EXC1	-0.49985	0.837116	0.198651	-0.02848	0.410869	-0.03035	-0.40468	-0.37036	0.814443	-0.14558
EXC2	0.61558	-0.23281	-0.61951	-0.43994	-0.86226	0.390898	0.786579	-0.26931	-0.27458	0.476333
EXC3	0.513103	-0.83095	-0.21396	0.043894	-0.41931	0.014102	0.415319	0.387983	-0.8113	0.165459
EYC1	-0.68738	0.286274	0.661515	0.485876	0.717284	-0.47879	-0.586	0.157605	0.278308	-0.20834
EYC2	0.454669	-0.5654	-0.28541	-0.14432	-0.55497	0.155506	0.57858	0.128355	-0.6742	0.05269
EYC3	0.672978	-0.28418	-0.64582	-0.48594	-0.69887	0.480583	0.566731	-0.15401	-0.27668	0.188
EZC1	-0.59939	-0.07623	0.784044	0.180531	0.883427	-0.10364	-0.86727	0.225039	0.004028	-0.70628
EZC2	0.245893	-0.07017	-0.3367	0.514077	-0.08988	-0.48022	0.129617	0.292886	0.030028	0.603218
EZC3	0.513942	0.043265	-0.69124	-0.19423	-0.82773	0.123585	0.804016	-0.23048	-0.04091	0.655005
EAC1	-0.48467	0.235399	0.50365	0.620653	0.624353	-0.6094	-0.48078	0.358299	0.26701	-0.10897
EAC2	-0.04547	-0.20188	0.06674	0.107559	-0.0806	-0.09415	0.187404	0.19621	-0.44715	-0.18235
EAC3	0.476887	-0.22617	-0.4973	-0.62122	-0.61228	0.610702	0.467756	-0.36023	-0.25612	0.098338
EBC1	-0.01483	-0.58945	0.229957	0.46325	0.182673	-0.3837	-0.07353	0.608256	-0.67956	-0.06781
EBC2	-0.74348	0.226911	0.689392	0.223055	0.863647	-0.18454	-0.81587	0.065211	0.202026	-0.60702
EBC3	0.029252	0.580282	-0.23582	-0.47153	-0.18931	0.393876	0.075952	-0.60821	0.681952	0.060569
ECC1	-0.62672	0.582388	0.472061	0.082217	0.544075	-0.09964	-0.53556	-0.01985	0.50958	-0.23231
ECC2	0.097292	-0.09837	-0.23776	-0.46559	-0.44895	0.45531	0.38029	-0.58705	-0.11674	0.064673
ECC3	0.559709	-0.47174	-0.46965	-0.18571	-0.65187	0.190581	0.65331	-0.07523	-0.49206	0.166952
ECC4	-0.32012	0.051009	0.272609	0.34897	0.521777	-0.3208	-0.48822	0.042205	0.111096	0.230908
W	-0.43716	0.237188	0.204376	0.090856	0.211116	-0.10889	-0.12512	-0.32917	0.090434	0.097696
T	-0.73862	0.52437	0.46905	0.123109	0.552563	-0.14373	-0.46449	-0.36693	0.414309	-0.23474



	EBB3	EBB4	ECB1	ECB2	ECB3	EXC1	EXC2	EXC3	EYC1	EYC2
EXB2	-0.16001	0.778494	-0.0842	0.87342	-0.03681	0.411343	-0.66681	-0.4272	0.676527	-0.40757
EXB3	0.439105	-0.34065	-0.5962	-0.24648	0.515624	-0.59373	0.41623	0.583141	-0.50479	0.40857
EYB1	-0.37551	-0.35824	-0.80081	-0.1461	0.869553	0.178753	0.26565	-0.18831	-0.47462	-0.25491
EYB2	0.52745	-0.27347	0.432009	0.036823	-0.39637	-0.5008	-0.04831	0.511437	0.016435	-0.05113
EYB3	0.233379	0.46011	0.849737	0.288482	-0.88527	-0.00343	-0.38955	0.013744	0.607527	0.053652
EZB1	0.265509	-0.77303	0.043981	-0.81666	0.02693	-0.49985	0.61558	0.513103	-0.68738	0.454669
EZB2	-0.82525	0.18913	0.207296	0.080924	-0.07643	0.837116	-0.23281	-0.83095	0.286274	-0.5654
EZB3	0.078342	0.745219	-0.12614	0.885745	-0.06004	0.198651	-0.61951	-0.21396	0.661515	-0.28541
EAB0	0.220164	0.227522	0.993393	0.242628	-0.93602	-0.02848	-0.43994	0.043894	0.485876	-0.14432
EAB1	-0.02734	0.715232	0.14881	0.997091	-0.29542	0.410869	-0.86226	-0.41931	0.717284	-0.55497
EAB2	-0.13943	-0.22671	-0.98242	-0.18597	0.902019	-0.03035	0.390898	0.014102	-0.47879	0.155506
EAB3	0.068356	-0.60553	0.047892	-0.95317	0.100606	-0.40468	0.786579	0.415319	-0.586	0.57858
EBB0	0.581379	-0.0617	0.734533	0.122162	-0.76632	-0.37036	-0.26931	0.387983	0.157605	0.128355
EBB1	-0.86817	0.133112	0.179984	0.090063	-0.05791	0.814443	-0.27458	-0.8113	0.278308	-0.6742
EBB2	-0.01421	-0.43972	0.439845	-0.60542	-0.19085	-0.14558	0.476333	0.165459	-0.20834	0.05269
EBB3	1	-0.05199	0.251401	0.015282	-0.38602	-0.8729	0.035723	0.874855	-0.08033	0.643165
EBB4	-0.05199	1	0.142753	0.746186	-0.24768	0.351048	-0.63462	-0.36704	0.892709	-0.31067
ECB1	0.251401	0.142753	1	0.187654	-0.93488	-0.07811	-0.39242	0.095537	0.421849	-0.13023
ECB2	0.015282	0.746186	0.187654	1	-0.34249	0.379432	-0.86881	-0.3877	0.748302	-0.52003
ECB3	-0.38602	-0.24768	-0.93488	-0.34249	1	0.136598	0.496654	-0.15086	-0.47659	0.007717
EXC1	-0.8729	0.351048	-0.07811	0.379432	0.136598	1	-0.37462	-0.99931	0.393541	-0.79976
EXC2	0.035723	-0.63462	-0.39242	-0.86881	0.496654	-0.37462	1	0.385578	-0.62982	0.537161
EXC3	0.874855	-0.36704	0.095537	-0.3877	-0.15086	-0.99931	0.385578	1	-0.39727	0.798197
EYC1	-0.08033	0.892709	0.421849	0.748302	-0.47659	0.393541	-0.62982	-0.39727	1	-0.47677
EYC2	0.643165	-0.31067	-0.13023	-0.52003	0.007717	-0.79976	0.537161	0.798197	-0.47677	1
EYC3	0.080451	-0.89016	-0.42276	-0.73056	0.472405	-0.38847	0.611559	0.392021	-0.99954	0.472376
EZC1	0.163367	0.641444	0.138961	0.892168	-0.33022	0.170211	-0.91353	-0.18727	0.569258	-0.35694
EZC2	0.13667	-0.23736	0.551005	-0.09905	-0.34113	-0.19702	-0.17721	0.200295	-0.04305	-0.25791
EZC3	-0.14061	-0.63984	-0.15441	-0.83959	0.342342	-0.19557	0.916336	0.213403	-0.55295	0.359998
EAC1	0.032118	0.820366	0.576883	0.664894	-0.60358	0.276779	-0.67092	-0.27966	0.945397	-0.45612
EAC2	0.4918	0.068401	0.061902	-0.04484	-0.24624	-0.43053	0.068229	0.428057	-0.10182	0.76542
EAC3	-0.04188	-0.81885	-0.57807	-0.65367	0.60227	-0.26398	0.656094	0.266685	-0.94416	0.444776
EBC1	0.898376	0.160805	0.457485	0.228092	-0.59149	-0.72325	-0.24755	0.721524	0.139152	0.535073
EBC2	-0.09314	0.638209	0.159843	0.85926	-0.26214	0.404068	-0.93848	-0.41983	0.515181	-0.47483
EBC3	-0.89648	-0.16543	-0.46486	-0.23492	0.59933	0.712599	0.238945	-0.71208	-0.14863	-0.53508
ECC1	-0.48362	0.46103	0.024795	0.527548	0.050218	0.669402	-0.40263	-0.66949	0.582061	-0.68245
ECC2	-0.13745	-0.3079	-0.49021	-0.47611	0.539495	-0.22387	0.385393	0.203721	-0.517	0.398897
ECC3	0.426143	-0.47064	-0.15968	-0.63466	0.067101	-0.6849	0.528464	0.680749	-0.64181	0.886881
ECC4	-0.09327	0.286555	0.356709	0.508339	-0.3018	0.357086	-0.37224	-0.34533	0.512579	-0.67109
W	-0.1924	0.45642	0.017123	0.212302	0.002194	0.280657	-0.15436	-0.29016	0.389734	-0.13105
T	-0.46076	0.639247	0.030228	0.545103	-0.08169	0.638456	-0.49626	-0.64917	0.572489	-0.40739



	EYC3	EZC1	EZC2	EZC3	EAC1	EAC2	EAC3	EBC1	EBC2	EBC3
EXB2	-0.65991	0.713223	-0.27591	-0.62551	0.488466	0.064787	-0.47975	0.09498	0.782326	-0.10459
EXB3	0.500921	-0.10326	-0.05095	0.125984	-0.50512	-0.06967	0.498342	0.123947	-0.33192	-0.11115
EYB1	0.478131	-0.11391	-0.13729	0.151333	-0.55506	-0.50176	0.559357	-0.58472	-0.08959	0.59636
EYB2	-0.01996	0.111275	0.662813	-0.08455	0.160347	-0.31434	-0.1643	0.417605	-0.17732	-0.40608
EYB3	-0.60859	0.186794	0.163368	-0.20479	0.659913	0.396481	-0.66226	0.502095	0.226437	-0.51775
EZB1	0.672978	-0.59939	0.245893	0.513942	-0.48467	-0.04547	0.476887	-0.01483	-0.74348	0.029252
EZB2	-0.28418	-0.07623	-0.07017	0.043265	0.235399	-0.20188	-0.22617	-0.58945	0.226911	0.580282
EZB3	-0.64582	0.784044	-0.3367	-0.69124	0.50365	0.06674	-0.4973	0.229957	0.689392	-0.23582
EAB0	-0.48594	0.180531	0.514077	-0.19423	0.620653	0.107559	-0.62122	0.46325	0.223055	-0.47153
EAB1	-0.69887	0.883427	-0.08988	-0.82773	0.624353	-0.0806	-0.61228	0.182673	0.863647	-0.18931
EAB2	0.480583	-0.10364	-0.48022	0.123585	-0.6094	-0.09415	0.610702	-0.3837	-0.18454	0.393876
EAB3	0.566731	-0.86727	0.129617	0.804016	-0.48078	0.187404	0.467756	-0.07353	-0.81587	0.075952
EBB0	-0.15401	0.225039	0.292886	-0.23048	0.358299	0.19621	-0.36023	0.608256	0.065211	-0.60821
EBB1	-0.27668	0.004028	0.030028	-0.04091	0.26701	-0.44715	-0.25612	-0.67956	0.202026	0.681952
EBB2	0.188	-0.70628	0.603218	0.655005	-0.10897	-0.18235	0.098338	-0.06781	-0.60702	0.060569
EBB3	0.080451	0.163367	0.13667	-0.14061	0.032118	0.4918	-0.04188	0.898376	-0.09314	-0.89648
EBB4	-0.89016	0.641444	-0.23736	-0.63984	0.820366	0.068401	-0.81885	0.160805	0.638209	-0.16543
ECB1	-0.42276	0.138961	0.551005	-0.15441	0.576883	0.061902	-0.57807	0.457485	0.159843	-0.46486
ECB2	-0.73056	0.892168	-0.09905	-0.83959	0.664894	-0.04484	-0.65367	0.228092	0.85926	-0.23492
ECB3	0.472405	-0.33022	-0.34113	0.342342	-0.60358	-0.24624	0.60227	-0.59149	-0.26214	0.59933
EXC1	-0.38847	0.170211	-0.19702	-0.19557	0.276779	-0.43053	-0.26398	-0.72325	0.404068	0.712599
EXC2	0.611559	-0.91353	-0.17721	0.916336	-0.67092	0.068229	0.656094	-0.24755	-0.93848	0.238945
EXC3	0.392021	-0.18727	0.200295	0.213403	-0.27966	0.428057	0.266685	0.721524	-0.41983	-0.71208
EYC1	-0.99954	0.569258	-0.04305	-0.55295	0.945397	-0.10182	-0.94416	0.139152	0.515181	-0.14863
EYC2	0.472376	-0.35694	-0.25791	0.359998	-0.45612	0.76542	0.444776	0.535073	-0.47483	-0.53508
EYC3	1	-0.55005	0.037746	0.53618	-0.94748	0.11043	0.946767	-0.13253	-0.49369	0.141981
EZC1	-0.55005	1	-0.0057	-0.98358	0.582837	0.01391	-0.56974	0.352574	0.880757	-0.33896
EZC2	0.037746	-0.0057	1	-0.02339	0.146548	-0.3842	-0.14686	0.212627	0.024755	-0.19365
EZC3	0.53618	-0.98358	-0.02339	1	-0.60107	0.013411	0.58774	-0.31417	-0.86115	0.296933
EAC1	-0.94748	0.582837	0.146548	-0.60107	1	-0.17043	-0.99974	0.222985	0.495434	-0.22359
EAC2	0.11043	0.01391	-0.3842	0.013411	-0.17043	1	0.168123	0.629986	0.033156	-0.64147
EAC3	0.946767	-0.56974	-0.14686	0.58774	-0.99974	0.168123	1	-0.22814	-0.48037	0.229002
EBC1	-0.13253	0.352574	0.212627	-0.31417	0.222985	0.629986	-0.22814	1	0.196963	-0.99874
EBC2	-0.49369	0.880757	0.024755	-0.86115	0.495434	0.033156	-0.48037	0.196963	1	-0.19198
EBC3	0.141981	-0.33896	-0.19365	0.296933	-0.22359	-0.64147	0.229002	-0.99874	-0.19198	1
ECC1	-0.57485	0.329813	-0.11314	-0.27498	0.455565	-0.26436	-0.44836	-0.29859	0.431597	0.290411
ECC2	0.516403	-0.31916	0.047838	0.326882	-0.59705	0.267706	0.597205	-0.07769	-0.2147	0.097684
ECC3	0.637113	-0.42417	-0.08151	0.391898	-0.59509	0.579939	0.588253	0.326081	-0.48645	-0.3179
ECC4	-0.51539	0.250229	0.380978	-0.24463	0.495491	-0.56171	-0.49343	-0.08865	0.239207	0.072331
W	-0.39329	0.078182	0.004156	-0.09179	0.272604	0.138955	-0.27228	-0.00829	0.204711	0.001701
T	-0.56455	0.371373	-0.18742	-0.36818	0.404765	0.09107	-0.39497	-0.1647	0.558981	0.154338

	ECC1	ECC2	ECC3	ECC4	W	T
EXB2	0.574453	-0.11826	-0.52448	0.28676	0.361939	0.675207
EXB3	-0.56425	0.302811	0.449368	-0.10418	-0.26485	-0.50257
EYB1	0.124038	0.316756	-0.14993	-0.08422	-0.22607	-0.1965
EYB2	-0.18696	-0.27434	-0.05521	0.433062	-0.28414	-0.47993
EYB3	0.066153	-0.38892	-0.05472	0.20719	0.276819	0.324227
EZB1	-0.62672	0.097292	0.559709	-0.32012	-0.43716	-0.73862
EZB2	0.582388	-0.09837	-0.47174	0.051009	0.237188	0.52437
EZB3	0.472061	-0.23776	-0.46965	0.272609	0.204376	0.46905
EAB0	0.082217	-0.46559	-0.18571	0.34897	0.090856	0.123109
EAB1	0.544075	-0.44895	-0.65187	0.521777	0.211116	0.552563
EAB2	-0.09964	0.45531	0.190581	-0.3208	-0.10889	-0.14373
EAB3	-0.53556	0.38029	0.65331	-0.48822	-0.12512	-0.46449
EBB0	-0.01985	-0.58705	-0.07523	0.042205	-0.32917	-0.36693
EBB1	0.50958	-0.11674	-0.49206	0.111096	0.090434	0.414309
EBB2	-0.23231	0.064673	0.166952	0.230908	0.097696	-0.23474
EBB3	-0.48362	-0.13745	0.426143	-0.09327	-0.1924	-0.46076
EBB4	0.46103	-0.3079	-0.47064	0.286555	0.45642	0.639247
ECB1	0.024795	-0.49021	-0.15968	0.356709	0.017123	0.030228
ECB2	0.527548	-0.47611	-0.63466	0.508339	0.212302	0.545103
ECB3	0.050218	0.539495	0.067101	-0.3018	0.002194	-0.08169
EXC1	0.669402	-0.22387	-0.6849	0.357086	0.280657	0.638456
EXC2	-0.40263	0.385393	0.528464	-0.37224	-0.15436	-0.49626
EXC3	-0.66949	0.203721	0.680749	-0.34533	-0.29016	-0.64917
EYC1	0.582061	-0.517	-0.64181	0.512579	0.389734	0.572489
EYC2	-0.68245	0.398897	0.886881	-0.67109	-0.13105	-0.40739
EYC3	-0.57485	0.516403	0.637113	-0.51539	-0.39329	-0.56455
EZC1	0.329813	-0.31916	-0.42417	0.250229	0.078182	0.371373
EZC2	-0.11314	0.047838	-0.08151	0.380978	0.004156	-0.18742
EZC3	-0.27498	0.326882	0.391898	-0.24463	-0.09179	-0.36818
EAC1	0.455565	-0.59705	-0.59509	0.495491	0.272604	0.404765
EAC2	-0.26436	0.267706	0.579939	-0.56171	0.138955	0.09107
EAC3	-0.44836	0.597205	0.588253	-0.49343	-0.27228	-0.39497
EBC1	-0.29859	-0.07769	0.326081	-0.08865	-0.00829	-0.1647
EBC2	0.431597	-0.2147	-0.48645	0.239207	0.204711	0.558981
EBC3	0.290411	0.097684	-0.3179	0.072331	0.001701	0.154338
ECC1	1	-0.25392	-0.87109	0.261382	0.272812	0.488742
ECC2	-0.25392	1	0.561652	-0.58167	0.200854	0.038304
ECC3	-0.87109	0.561652	1	-0.60922	-0.13943	-0.36984
ECC4	0.261382	-0.58167	-0.60922	1	0.182422	0.246508
W	0.272812	0.200854	-0.13943	0.182422	1	0.829152
T	0.488742	0.038304	-0.36984	0.246508	0.829152	1

## Appendix VI The coefficients of the fitted curves

Table F-1-a: The elements of the matrix PA from EXX2 to ECZ1

	p1	p2	p3	p4
EXX2	-3.99E-15	5.94E-12	-2.01E-09	1.25E-07
EXX3	-3.44E-17	5.01E-14	-1.59E-11	1.10E-09
EYX2	-1.29E-15	1.82E-12	-7.86E-10	2.46E-07
EYX3	1.68E-17	-2.15E-14	3.67E-12	9.79E-10
EZX2	8.83E-16	-6.24E-12	4.53E-09	-2.89E-07
EZX3	-2.93E-17	3.72E-14	-1.15E-11	1.43E-09
EAX0	9.13E-12	-1.21E-08	3.86E-06	-4.57E-04
EAX1	-2.80E-17	1.05E-13	-5.43E-11	7.69E-09
EAX2	-1.34E-17	1.54E-14	-3.39E-12	9.21E-10
EAX3	8.29E-21	-2.08E-17	1.10E-14	-9.63E-13
EBX1	-2.74E-15	4.68E-12	-1.71E-09	2.76E-08
EBX2	3.15E-17	-4.01E-14	1.30E-11	-1.45E-09
EBX3	4.88E-20	-4.96E-17	3.53E-15	2.75E-12
ECX0	-2.36E-11	3.42E-08	-1.08E-05	-1.79E-04
ECX1	5.61E-16	-5.05E-13	-7.37E-12	-1.99E-08
ECX2	1.60E-17	-2.15E-14	4.80E-12	6.30E-10
ECX3	5.91E-20	-9.15E-17	3.41E-14	-8.04E-13
EXY0	7.19E-10	-1.08E-06	3.45E-04	-1.98E-01
EXY2	1.64E-16	3.66E-14	-1.67E-10	4.43E-08
EXY3	1.67E-17	-2.31E-14	1.00E-11	-2.68E-09
EYY0	-2.97E-10	4.82E-07	-1.98E-04	3.90E-02
EYY1	-1.32E-12	1.53E-09	-1.91E-07	1.69E-04
EYY2	9.39E-16	-1.93E-12	1.15E-09	-2.13E-07
EYY3	-6.22E-18	1.38E-14	-8.46E-12	-3.32E-10
EZY2	-1.40E-15	3.91E-12	-2.19E-09	-9.01E-08
EZY3	3.76E-18	-6.45E-18	-4.60E-12	5.16E-10
EXZ1	4.01E-12	-5.87E-09	2.15E-06	-2.30E-04
EXZ2	-8.05E-15	1.22E-11	-5.33E-09	9.66E-07
EXZ3	2.08E-19	1.79E-16	1.33E-12	-1.11E-09
EYZ2	-1.45E-14	1.75E-11	-3.37E-09	-1.36E-07
EYZ3	2.22E-17	-2.11E-14	-2.91E-14	3.44E-10
EZZ1	2.39E-12	-3.67E-09	1.16E-06	1.45E-04
EZZ2	-2.09E-14	3.07E-11	-8.99E-09	3.32E-07
EZZ3	2.74E-17	-3.45E-14	5.99E-12	-1.44E-09
EAZ0	5.08E-12	-6.76E-09	1.51E-06	1.45E-04
EAZ1	-4.65E-14	5.76E-11	-7.27E-09	-2.60E-06
EAZ2	2.25E-16	-2.98E-13	5.16E-11	9.53E-09
EAZ3	-2.99E-19	4.55E-16	-1.16E-13	-9.43E-12
ECZ0	-1.01E-12	3.24E-09	-2.63E-06	5.37E-04
ECZ1	1.69E-14	-5.48E-11	4.50E-08	-9.70E-06

Table F-1-b: The elements of the matrix PA from ECZ2 to ECC4

	p1	p2	p3	p4
ECZ2	-2.29E-17	2.04E-13	-2.17E-10	5.49E-08
ECZ3	-6.76E-20	-1.96E-16	3.22E-13	-9.94E-11
EXB0	8.03E-10	-1.35E-06	4.79E-04	-2.06E-01
EXB1	2.28E-09	-3.70E-06	1.51E-03	-1.91E-01
EXB2	-5.06E-10	8.02E-07	-2.76E-04	4.07E-02
EXB3	-4.93E-11	1.14E-07	-5.44E-05	1.73E-02
EYB1	1.40E-10	-2.10E-07	8.99E-05	-1.44E-02
EYB2	-4.80E-11	5.91E-08	-1.52E-05	-7.21E-04
EYB3	-1.86E-10	2.74E-07	-1.11E-04	1.13E-02
EZB1	9.36E-10	-1.52E-06	5.29E-04	-1.05E-01
EZB2	1.21E-09	-1.93E-06	7.86E-04	-8.69E-02
EZB3	-2.35E-10	3.62E-07	-1.24E-04	1.62E-02
EAB0	-6.25E-12	8.23E-09	-2.76E-06	4.19E-04
EAB1	-2.33E-11	3.37E-08	-1.05E-05	-2.23E-04
EAB2	2.22E-12	-2.87E-09	9.66E-07	-1.58E-04
EAB3	3.06E-12	-4.37E-09	1.26E-06	5.75E-05
EBB0	-7.80E-13	8.70E-10	-2.56E-07	1.14E-05
EBB1	6.62E-13	-1.14E-09	5.17E-07	-4.28E-05
EBB2	4.23E-13	-6.88E-10	2.45E-07	-3.36E-06
EBB3	-1.20E-12	1.82E-09	-7.40E-07	4.72E-05
EBB4	-1.12E-13	1.67E-10	-5.46E-08	3.36E-06
ECB1	-5.59E-12	7.22E-09	-2.39E-06	4.18E-04
ECB2	-1.09E-11	1.57E-08	-4.97E-06	-5.44E-05
ECB3	1.05E-12	-1.45E-09	5.32E-07	-9.00E-05
EXC1	5.26E-11	-8.41E-08	3.83E-05	-2.63E-03
EXC2	2.43E-11	-3.31E-08	9.48E-06	6.49E-06
EXC3	-5.14E-12	8.24E-09	-3.81E-06	2.72E-04
EYC1	-1.23E-10	1.71E-07	-5.01E-05	3.24E-03
EYC2	7.03E-13	3.67E-09	-5.51E-06	8.03E-04
EYC3	1.21E-11	-1.68E-08	4.89E-06	-3.23E-04
EZC1	-3.78E-11	5.27E-08	-1.58E-05	4.43E-04
EZC2	-2.98E-12	3.40E-09	-5.82E-07	-7.58E-05
EZC3	2.89E-12	-3.96E-09	1.15E-06	-2.31E-05
EAC1	-6.94E-13	9.11E-10	-2.45E-07	7.19E-06
EAC2	-1.63E-14	3.87E-11	-2.63E-08	1.94E-06
EAC3	7.06E-14	-9.27E-11	2.50E-08	-7.68E-07
EBC1	-3.94E-13	5.96E-10	-2.35E-07	1.07E-05
EBC2	-6.23E-14	8.91E-11	-2.71E-08	4.96E-07
EBC3	3.96E-14	-6.04E-11	2.41E-08	-1.10E-06
ECC1	1.20E-14	-3.20E-11	2.36E-08	-4.54E-06
ECC2	1.26E-14	-1.37E-11	2.95E-09	-4.74E-06
ECC3	1.94E-15	2.86E-13	-2.74E-09	4.26E-07
ECC4	-1.16E-13	1.58E-10	-4.06E-08	-2.92E-05

Table F-2\_a: The elements of the matrix PB from EXX2 to ECZ1

	p1	p2	p3	p4
EXX2	-1.24E-12	1.92E-10	-5.09E-09	5.84E-08
EXX3	-7.82E-15	5.66E-13	5.26E-11	-6.40E-10
EYX2	-1.26E-12	2.76E-10	-1.69E-08	3.38E-07
EYX3	4.96E-15	-3.50E-13	-4.41E-11	1.72E-09
EZX2	1.13E-12	-4.12E-10	2.56E-08	2.53E-08
EZX3	-1.02E-14	1.42E-12	-4.86E-11	1.36E-09
EAX0	3.89E-09	-6.52E-07	2.80E-05	-5.24E-04
EAX1	8.81E-14	-1.84E-11	1.07E-09	-2.29E-09
EAX2	-5.81E-15	9.77E-13	-4.49E-11	1.44E-09
EAX3	-1.66E-18	3.72E-16	-3.43E-14	2.99E-13
EBX1	-1.10E-12	2.18E-10	-8.21E-09	-1.59E-08
EBX2	1.15E-14	-1.61E-12	5.73E-11	-1.27E-09
EBX3	1.47E-17	-1.42E-15	-2.40E-14	1.91E-12
ECX0	-7.71E-09	1.17E-06	-2.73E-05	-5.22E-04
ECX1	1.69E-13	-1.78E-11	1.20E-10	-4.46E-08
ECX2	4.84E-15	-4.25E-13	-3.00E-11	1.33E-09
ECX3	2.03E-17	-3.22E-15	9.78E-14	8.25E-13
EXY0	2.15E-07	-3.29E-05	6.07E-04	-1.85E-01
EXY2	-5.66E-14	2.95E-11	-2.45E-09	4.09E-08
EXY3	9.10E-15	-1.63E-12	8.99E-11	-2.66E-09
EYY0	-1.65E-07	3.64E-05	-2.10E-03	4.67E-02
EYY1	4.54E-10	-2.23E-07	2.13E-05	-7.31E-05
EYY2	9.33E-13	-2.39E-10	1.66E-08	-2.70E-07
EYY3	-1.03E-14	2.96E-12	-2.19E-10	1.31E-09
EZY2	-1.04E-12	2.47E-10	-1.11E-08	-2.69E-07
EZY3	4.32E-16	2.12E-13	-3.87E-11	2.39E-10
EXZ1	1.37E-09	-2.16E-07	8.04E-06	-1.68E-04
EXZ2	-2.58E-12	4.02E-10	-1.94E-08	7.61E-07
EXZ3	-1.08E-15	3.12E-13	-8.24E-12	-9.03E-10
EYZ2	-4.11E-12	5.23E-10	-6.73E-09	8.96E-08
EYZ3	4.99E-15	-3.74E-13	-1.53E-11	-5.46E-10
EZZ1	1.09E-09	-2.59E-07	1.50E-05	5.47E-07
EZZ2	-3.06E-12	7.16E-11	4.49E-08	-5.19E-07
EZZ3	-4.63E-15	2.47E-12	-2.44E-10	8.11E-10
EAZ0	1.12E-09	-1.05E-07	-5.30E-06	2.04E-04
EAZ1	-5.40E-12	-5.85E-10	1.85E-07	-3.80E-06
EAZ2	2.49E-14	2.57E-12	-8.72E-10	1.73E-08
EAZ3	-2.59E-17	-3.90E-15	1.22E-12	-2.67E-11
ECZ0	-1.05E-09	3.30E-07	-2.81E-05	5.63E-04
ECZ1	1.89E-11	-5.72E-09	4.75E-07	-9.84E-06

Table F-2\_b: The elements of the matrix PB from ECZ2 to ECC4

	p1	p2	p3	p4
ECZ2	-7.76E-14	2.73E-11	-2.47E-09	5.59E-08
ECZ3	9.02E-17	-4.02E-14	4.07E-12	-1.04E-10
EXB0	2.53E-07	-4.76E-05	1.52E-03	-1.92E-01
EXB1	1.05E-06	-2.17E-04	1.16E-02	-2.04E-01
EXB2	-1.61E-07	2.81E-05	-8.53E-04	3.31E-02
EXB3	-2.67E-08	7.98E-06	-5.10E-04	1.88E-02
EYB1	7.12E-08	-1.33E-05	7.06E-04	-1.37E-02
EYB2	-1.78E-08	2.97E-06	-1.35E-04	7.41E-04
EYB3	-8.80E-08	1.61E-05	-8.10E-04	1.05E-02
EZB1	2.92E-07	-5.30E-05	1.64E-03	-9.12E-02
EZB2	5.60E-07	-1.15E-04	6.10E-03	-9.29E-02
EZB3	-7.03E-08	1.15E-05	-3.26E-04	1.27E-02
EAB0	-2.92E-09	4.99E-07	-2.31E-05	4.68E-04
EAB1	-7.50E-09	1.12E-06	-2.45E-05	-5.63E-04
EAB2	1.08E-09	-1.83E-07	8.59E-06	-1.77E-04
EAB3	8.66E-10	-1.18E-07	8.61E-07	1.07E-04
EBB0	-3.18E-10	3.92E-08	-1.25E-06	5.74E-06
EBB1	3.41E-10	-7.34E-08	4.15E-06	-4.16E-05
EBB2	8.72E-11	-5.97E-09	-8.44E-07	2.73E-05
EBB3	-5.76E-10	1.10E-07	-5.62E-06	4.48E-05
EBB4	-4.14E-11	6.75E-09	-1.89E-07	1.47E-06
ECB1	-2.62E-09	4.42E-07	-2.06E-05	4.72E-04
ECB2	-3.58E-09	5.41E-07	-1.28E-05	-2.15E-04
ECB3	4.60E-10	-7.81E-08	3.60E-06	-8.73E-05
EXC1	3.09E-08	-6.63E-06	4.02E-04	-3.59E-03
EXC2	9.72E-09	-1.47E-06	3.65E-05	2.88E-04
EXC3	-3.03E-09	6.54E-07	-4.01E-05	3.69E-04
EYC1	-3.77E-08	5.48E-06	-1.23E-04	2.45E-03
EYC2	-2.18E-09	7.40E-07	-6.61E-05	7.21E-04
EYC3	3.70E-09	-5.38E-07	1.20E-05	-2.52E-04
EZC1	-1.42E-08	2.15E-06	-5.37E-05	-5.07E-05
EZC2	-1.67E-09	3.05E-07	-1.37E-05	7.39E-05
EZC3	1.13E-09	-1.67E-07	3.81E-06	1.89E-05
EAC1	-2.46E-10	3.51E-08	-7.99E-07	4.16E-06
EAC2	-1.25E-11	3.04E-09	-2.10E-07	1.03E-06
EAC3	2.51E-11	-3.60E-09	8.38E-08	-4.77E-07
EBC1	-1.90E-10	3.61E-08	-1.80E-06	1.04E-05
EBC2	-2.59E-11	4.08E-09	-1.04E-07	-5.92E-07
EBC3	1.90E-11	-3.63E-09	1.83E-07	-1.06E-06
ECC1	1.78E-11	-4.48E-09	3.05E-07	-4.53E-06
ECC2	1.33E-12	3.92E-10	-5.52E-08	-4.08E-06
ECC3	-1.40E-12	5.24E-10	-4.68E-08	4.50E-07
ECC4	-3.39E-11	5.25E-09	-1.37E-07	-2.80E-05

Table F-3\_a: The elements of the matrix P from EXX2 to EAB1

	p00	p10	p01	p20	p11	p02	p30	p21	p12	p03
EXX2	7.51E-08	-7.75E-10	-6.52E-10	2.00E-12	-7.18E-12	9.78E-11	-7.54E-15	2.34E-13	-2.71E-12	9.62E-12
EXX3	-1.04E-10	-6.09E-12	1.46E-10	5.68E-15	-4.18E-13	2.01E-12	2.12E-17	-2.41E-16	1.45E-15	-4.49E-15
EYX2	3.05E-07	-6.95E-10	-1.58E-09	2.12E-12	3.57E-11	-4.40E-10	-1.51E-14	4.19E-13	-4.44E-12	1.68E-11
EYX3	1.42E-09	7.38E-12	-1.50E-10	-2.12E-14	1.60E-13	1.51E-12	2.53E-17	-4.63E-16	4.64E-15	-2.86E-14
EZX2	-1.67E-07	7.20E-09	-6.39E-09	-2.37E-11	3.08E-10	-2.40E-09	-7.50E-15	6.67E-13	-9.90E-12	4.89E-11
EZX3	1.38E-09	6.76E-12	-4.63E-11	-3.57E-14	5.27E-13	-3.27E-12	-4.24E-17	2.39E-15	-3.30E-14	1.42E-13
EAX0	-0.00047893	7.76E-07	-2.41E-06	-4.49E-09	-1.65E-08	8.22E-07	2.89E-11	-9.21E-10	1.05E-08	-4.38E-08
EAX1	1.32E-08	1.97E-09	-1.48E-08	-8.22E-12	7.85E-11	-7.43E-11	1.03E-14	-1.94E-13	1.46E-12	-4.79E-12
EAX2	1.20E-09	1.29E-11	-8.32E-11	-7.67E-14	1.35E-12	-6.93E-12	1.92E-19	1.16E-15	-2.26E-14	1.13E-13
EAX3	-7.36E-13	-7.60E-14	5.90E-13	3.40E-16	-3.09E-15	4.39E-16	-5.18E-19	1.08E-17	-8.91E-17	3.09E-16
EBX1	-3.55E-09	-3.46E-09	9.19E-09	1.81E-11	-2.05E-10	9.09E-10	-2.12E-14	3.69E-13	-2.31E-12	3.63E-12
EBX2	-1.34E-09	4.44E-13	-9.84E-12	4.81E-15	-1.35E-13	2.49E-12	7.31E-17	-3.01E-15	3.75E-14	-1.57E-13
EBX3	2.51E-12	-1.25E-14	2.16E-14	-4.81E-17	5.91E-16	1.61E-15	2.54E-19	-7.59E-18	8.04E-17	-3.10E-16
ECX0	-0.000400354	6.07E-06	-5.08E-05	-3.81E-08	5.42E-07	-1.53E-06	3.45E-11	-3.69E-10	-2.04E-09	1.99E-08
ECX1	-3.05E-08	-1.00E-09	2.79E-09	4.98E-12	-7.45E-11	4.33E-10	-1.58E-15	-2.51E-14	8.23E-13	-5.21E-12
ECX2	1.07E-09	7.78E-12	-1.18E-10	-4.21E-14	6.23E-13	-1.56E-12	3.56E-17	-5.47E-16	2.35E-15	-5.98E-15
ECX3	2.54E-13	4.16E-14	-1.44E-13	-1.83E-16	2.36E-15	-1.15E-14	1.57E-19	-2.92E-18	2.09E-17	-3.50E-17
EXY0	-0.185718245	-0.000502707	0.001793755	3.43E-06	-4.60E-05	0.000167141	-4.97E-09	1.02E-07	-6.86E-07	1.42E-06
EXY2	4.75E-08	-1.11E-09	2.55E-09	6.04E-12	-7.72E-11	3.50E-10	-5.76E-15	8.26E-14	-2.32E-13	-1.05E-12
EXY3	-2.69E-09	2.63E-11	-1.17E-10	-1.67E-13	2.33E-12	-7.78E-12	2.00E-16	-4.22E-15	3.02E-14	-8.08E-14
EY0	0.04433104	-0.000219321	-0.000502754	1.01E-06	-5.76E-06	8.64E-06	-1.92E-09	4.36E-08	-4.14E-07	1.44E-06
EY1	7.07E-06	1.49E-06	1.77E-05	-1.14E-08	6.71E-08	-1.95E-07	1.77E-11	-3.18E-10	2.44E-09	-6.95E-09
EY2	-2.58E-07	1.63E-09	3.48E-09	-7.63E-12	6.24E-11	-1.97E-10	1.21E-14	-2.54E-13	2.10E-12	-6.19E-12
EY3	7.79E-10	-2.02E-11	-1.25E-10	1.33E-13	-1.10E-12	4.14E-12	-1.91E-16	3.61E-15	-2.79E-14	7.67E-14
EZY2	-1.48E-07	-2.12E-09	1.56E-09	6.65E-12	-1.05E-10	9.24E-10	7.58E-15	-3.65E-13	4.84E-12	-2.23E-11
EZY3	4.28E-10	-1.50E-11	2.09E-11	6.93E-14	-1.06E-12	5.81E-12	-1.88E-17	-2.42E-16	9.33E-15	-5.88E-14
EXZ1	-0.000187914	9.87E-07	6.52E-07	-3.45E-09	3.70E-08	-1.85E-07	1.09E-11	-3.46E-10	3.79E-09	-1.33E-08
EXZ2	8.29E-07	-4.34E-09	1.57E-08	3.67E-12	1.90E-11	-2.96E-10	-2.55E-14	9.45E-13	-1.13E-11	4.28E-11
EXZ3	-9.31E-10	2.20E-12	-5.38E-11	4.22E-14	-7.64E-13	4.55E-12	-1.64E-17	-3.71E-16	1.08E-14	-5.99E-14
EYZ2	-8.42E-08	9.59E-09	-5.54E-08	-3.96E-11	6.30E-10	-3.18E-09	-5.20E-15	8.06E-13	-1.42E-11	6.92E-11
EYZ3	-5.47E-11	-3.93E-11	2.25E-10	1.59E-13	-2.18E-12	9.64E-12	-4.88E-17	-9.45E-16	2.79E-14	-1.52E-13
EZZ1	5.71E-05	1.81E-07	2.03E-05	-4.39E-09	-9.72E-09	3.96E-08	9.65E-12	-1.98E-10	2.11E-09	-8.39E-09
EZZ2	-2.79E-07	8.53E-09	-1.06E-08	-5.07E-11	4.37E-10	-5.53E-10	8.07E-14	-1.54E-12	1.05E-11	-2.65E-11
EZZ3	3.22E-10	-3.63E-11	-2.89E-11	2.02E-13	-1.66E-12	4.24E-12	-2.86E-16	5.14E-15	-3.55E-14	8.82E-14
EAZ0	0.000206169	-2.02E-06	1.06E-05	5.77E-09	-5.65E-08	6.76E-08	-2.42E-12	-3.26E-11	1.20E-09	-6.09E-09
EAZ1	-3.58E-06	3.39E-08	-9.71E-08	-1.06E-10	8.26E-10	-5.62E-10	9.23E-14	-1.15E-12	1.72E-12	1.06E-11
EAZ2	1.52E-08	-1.42E-10	3.73E-10	5.16E-13	-4.21E-12	8.16E-13	-5.20E-16	7.96E-15	-3.49E-14	6.61E-14
EAZ3	-2.08E-11	5.25E-14	2.38E-13	-2.58E-16	-4.32E-16	3.29E-14	6.72E-19	-1.64E-17	1.55E-16	-6.50E-16
ECZ0	0.000560553	-3.58E-06	-8.61E-06	1.20E-08	-1.14E-07	7.26E-07	-1.59E-11	3.48E-10	-2.81E-09	5.54E-09
ECZ1	-1.00E-05	7.68E-08	4.36E-09	-2.55E-10	2.39E-09	-1.17E-08	3.31E-13	-7.06E-12	5.63E-11	-1.29E-10
ECZ2	5.72E-08	-4.30E-10	2.26E-10	1.14E-12	-7.41E-12	3.21E-11	-1.57E-15	3.38E-14	-2.92E-13	7.71E-13
ECZ3	-1.06E-10	6.72E-13	-3.08E-13	-7.17E-16	-1.07E-14	6.21E-14	1.68E-18	-4.31E-17	5.07E-16	-1.85E-15
EXB0	-0.194079883	-0.000501561	0.003938087	3.54E-06	-4.97E-05	0.000125592	-5.43E-09	1.21E-07	-8.85E-07	2.46E-06
EXB1	-0.201108604	-0.000699298	0.017663801	2.61E-06	-3.57E-05	-0.000148868	-2.96E-09	4.29E-08	-5.34E-08	7.56E-07
EXB2	0.033996892	0.0001896	-0.001750971	-1.31E-06	1.85E-05	-3.99E-05	1.87E-09	-3.90E-08	2.49E-07	-6.20E-07
EXB3	0.018669842	0.000121425	-0.001858555	-4.23E-07	4.42E-06	1.95E-05	6.71E-10	-1.60E-08	1.42E-07	-6.39E-07
EYB1	-0.014295371	0.00015479	-0.000651156	-7.27E-07	8.29E-06	-1.97E-05	1.06E-09	-2.45E-08	2.10E-07	-6.97E-07
EYB2	-0.000162502	2.56E-05	-0.000274685	-6.50E-08	1.09E-06	-4.70E-06	-8.33E-11	3.98E-09	-5.45E-08	2.28E-07
EYB3	0.010959865	-0.000138698	0.000593474	6.32E-07	-6.95E-06	1.24E-05	-9.88E-10	2.41E-08	-2.20E-07	7.82E-07
EZB1	-0.092133787	-0.000514942	0.004188603	3.48E-06	-4.87E-05	0.000119866	-5.22E-09	1.14E-07	-8.02E-07	2.13E-06
EZB2	-0.091712413	-0.00023671	0.007974525	8.60E-07	-1.36E-05	-7.24E-05	-8.03E-10	5.54E-09	1.20E-07	-1.86E-07
EZB3	0.01306203	6.70E-05	-0.000755324	-3.86E-07	4.73E-06	-1.74E-06	5.43E-10	-1.00E-08	5.26E-08	-1.32E-07
EAB0	0.000442588	-1.45E-06	8.36E-06	7.47E-09	-4.58E-08	-4.78E-07	-2.50E-11	7.46E-10	-8.05E-09	3.28E-08
EAB1	-0.000443977	6.65E-06	-5.22E-05	-4.12E-08	5.85E-07	-1.70E-06	3.75E-11	-4.26E-10	-1.71E-09	1.94E-08

Table F-3\_b: The elements of the matrix Pfrom EAB2 to ECC4

	p00	p10	p01	p20	p11	p02	p30	p21	p12	p03
EAB2	-0.000167891	6.50E-07	-4.96E-06	-3.49E-09	2.79E-08	1.79E-07	1.03E-11	-3.00E-10	3.15E-09	-1.28E-08
EAB3	9.14E-05	-1.52E-06	1.03E-05	8.40E-09	-1.11E-07	2.92E-07	-8.63E-12	1.30E-10	-3.01E-10	-9.57E-10
EBB0	7.37E-06	-2.41E-07	3.75E-06	1.02E-09	-1.22E-08	-7.15E-08	-3.44E-12	1.15E-10	-1.27E-09	5.29E-09
EBB1	-4.50E-05	-1.07E-07	4.50E-06	1.18E-09	-1.95E-08	2.91E-09	-2.33E-12	5.06E-11	-3.36E-10	1.08E-09
EBB2	1.56E-05	1.13E-07	-1.51E-06	-1.34E-11	5.34E-09	-6.24E-08	-6.29E-13	1.51E-11	-1.59E-10	8.47E-10
EBB3	4.76E-05	-2.01E-07	-2.28E-06	4.98E-10	-3.18E-10	1.55E-08	-1.14E-12	4.23E-11	-5.59E-10	2.18E-09
EBB4	2.10E-06	-3.95E-08	1.38E-07	2.19E-10	-2.70E-09	1.19E-08	-2.11E-13	3.72E-12	-2.68E-11	6.53E-11
ECB1	0.000446237	-9.11E-07	5.32E-06	5.85E-09	-3.75E-08	-4.47E-07	-2.30E-11	7.06E-10	-7.66E-09	3.11E-08
ECB2	-0.000157682	2.32E-06	-1.99E-05	-1.48E-08	2.13E-07	-6.01E-07	1.24E-11	-9.65E-11	-1.46E-09	1.06E-08
ECB3	-8.92E-05	4.10E-07	-1.55E-06	-2.06E-09	2.20E-08	-1.41E-08	4.79E-12	-1.35E-10	1.36E-09	-5.06E-09
EXC1	-0.003071208	1.56E-05	0.000173401	-6.94E-08	4.40E-07	-2.72E-06	1.38E-10	-3.75E-09	3.94E-08	-1.30E-07
EXC2	0.000150892	2.91E-06	-3.29E-05	-5.54E-09	-1.29E-07	1.78E-06	4.65E-11	-1.48E-09	1.80E-08	-7.63E-08
EXC3	0.000317397	-1.37E-06	-1.86E-05	6.05E-09	-3.66E-08	2.54E-07	-1.27E-11	3.58E-10	-3.83E-09	1.28E-08
EYC1	0.002144005	-1.84E-05	3.19E-05	1.83E-07	-2.49E-06	9.34E-06	-2.80E-10	7.03E-09	-6.78E-08	2.40E-07
EYC2	0.000796555	-1.43E-05	3.59E-05	5.44E-08	-6.59E-07	2.83E-06	-4.47E-11	6.80E-10	-2.24E-09	-7.18E-09
EYC3	-0.000217728	1.93E-06	-3.30E-06	-1.94E-08	2.66E-07	-1.01E-06	2.91E-11	-7.19E-10	6.81E-09	-2.37E-08
EZC1	0.000137312	-6.00E-06	3.45E-05	1.91E-08	-6.29E-09	-9.02E-07	-6.96E-11	2.10E-09	-2.48E-08	9.85E-08
EZC2	4.74E-06	1.30E-06	-1.05E-05	-4.67E-09	1.39E-07	-9.40E-07	-8.19E-12	2.99E-10	-4.17E-09	1.94E-08
EZC3	-1.48E-06	9.47E-07	-6.06E-06	-4.13E-09	3.22E-08	-8.86E-09	9.02E-12	-2.36E-10	2.46E-09	-9.14E-09
EAC1	3.21E-06	-1.78E-07	1.10E-06	1.53E-09	-1.95E-08	4.92E-08	-2.66E-12	6.63E-11	-6.19E-10	2.19E-09
EAC2	1.54E-06	-4.87E-08	2.02E-07	1.23E-10	-1.07E-09	2.79E-09	-8.13E-14	1.16E-12	-6.30E-12	1.10E-11
EAC3	-3.69E-07	1.78E-08	-1.07E-07	-1.56E-10	2.00E-09	-5.17E-09	2.69E-13	-6.67E-12	6.22E-11	-2.20E-10
EBC1	1.04E-05	-9.83E-08	-2.35E-07	1.89E-10	1.35E-09	-1.37E-08	-5.39E-13	1.74E-11	-2.23E-10	9.30E-10
EBC2	-2.23E-08	2.31E-09	3.91E-08	-7.35E-11	1.56E-09	-8.25E-09	3.47E-14	7.75E-14	-1.51E-11	9.97E-11
EBC3	-1.07E-06	8.86E-09	3.27E-08	-1.27E-11	-1.99E-10	1.45E-09	4.29E-14	-1.50E-12	2.04E-11	-8.69E-11
ECC1	-5.20E-06	3.04E-08	1.13E-07	-1.83E-10	2.44E-09	-1.29E-08	2.61E-13	-5.44E-12	3.59E-11	-5.07E-11
ECC2	-4.47E-06	-1.41E-08	4.48E-08	3.90E-11	5.76E-11	-1.26E-09	-4.44E-14	2.65E-13	3.89E-13	-7.28E-12
ECC3	5.02E-07	-1.01E-08	2.04E-08	4.40E-11	-5.05E-10	2.06E-09	-5.18E-14	9.44E-13	-5.47E-12	6.35E-12
ECC4	-2.81E-05	1.44E-07	-1.27E-06	-5.40E-10	6.47E-09	-1.34E-08	5.08E-13	-7.42E-12	2.08E-11	3.08E-11



**Appendix VII-Figure G-1: The error modeling as a function of weight and torque**

